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ABSTRACT

Provided are the papers presented at a conference which served as an international forum on diagnostic and prescriptive mathematics education. They are: (1) "The Evolution of the Research Council for Diagnostic and Prescriptive Mathematics" by Robert Underhill; (2) "The Interaction of Knowledge and Cognitive Processes in Diagnosis and Prescription--Implications from Artificial Intelligence Research" by Tom Cooper and Rod Nason; (3) "Mastering Subtraction Facts: Implications from American and Australian Research" by Carol Thornton, Margaret Toohey, and Graham Jones; (4) "Interviews with Math Anxious Adults" by Carole Bauer; (5) "Teacher Training for Diagnosis and Prescription in Mathematics Education" by Kathryn Crawford; (6) "Project in the Mathematical Education of Re-entry Adult Learners" by Leone Burton; (7) "Aid for Problem Solving Diagnosis and Prescription in Action at Year 12" by Tom Cooper and Lyn English; (8) "A Systematic Approach to Developing Instructional Materials Focusing on Mathematical Word Problems" by E. Ray Phillips and A. Edward Uprichard; (9) "Qualitative Mathematics--The First Priority" by Douglas Crawford; (10) "Difference, Dividing, and Multiplication Strategies in Two Kinds of Fraction Problems" by George Bright and John Harvey; and (11) "Research on Learning and Instruction: Variables and Methods" by A. Edward Uprichard. (JN)

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RESEARCH REPORTS
FROM

THE FIRST PRE-ICME SATELLITE CONFERENCE ON DIAGNOSTIC & PRESCRIPTIVE MATHEMATICS

1984

EDITED BY
DUDLEY BLANE
MONASH UNIVERSITY

1984 AUSTRALIAN CONFERENCE

PRESIDENT:	Dr Bob Underhill Virginia Polytechnic Institute and State University U.S.A.
CONFERENCE CO-DIRECTOR AND PROGRAM CHAIR:	Dr Jon M. Engelhardt Arizona State University U.S.A.
CONFERENCE CO-DIRECTOR AND AUSTRALIAN ORGANISER:	Dr Dudley Blane Monash University Australia
LOCAL ORGANISERS AND HELPERS:	Dr Gilah Leder Ms Foong Pui Yee Ms Maria Papamichael Ms Carla Whiting Mr Rodney Bate Mr Chris Evangelou Mr Robert Meiselbach All Monash University

PREFACE

The conference held at Monash University, Australia, was organised under the auspices of the Research Council for Diagnostic and Prescriptive Mathematics as one of the pre-conferences to the Fifth International Congress on Mathematical Education (ICME5). Designed as a meeting to provide an international forum on diagnostic and prescriptive mathematics education it attracted twenty three speakers and nearly one hundred partcipants from many countries including Australia, Canada, England, Germany, Papua New Guinea, South Africa and the United States of America.

A number of speakers at the Conference provided papers designed to give a record of some of the proceedings, both for those attending and others who might be interested. These papers have been assembled and reproduced in this monograph. In the interests of cost they have not been re-typed and to a large extent are in their original format with editing kept to a minimum. I apologise for any imperfections this may have caused and hope that the reader will agree that the lack of standardisation in type setting and layout is more than outweighed by bringing to your attention the significant thoughts, ideas, premises, research hypotheses and conclusions of the conference contributors as quickly and cheaply as possible.

Dudley Blane
Mathematics Education Centre
Monash University.

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THE EVOLUTION OF THE RESEARCH COUNCIL FOR DIAGNOSTIC AND PRESCRIPTIVE MATHEMATICS

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This topic is both a challenge and an opportunity. I have occasionally asked myself where the diagnostic/prescriptive movement fits and what its boundaries are, but preparing this presentation created the need to explore the relationship in detail.

This presentation is in three sections. First, I will briefly discuss historical evolution of mathematical diagnosis and remediation, then I will discuss the RCDPM mission and how the organization sought to implement it. Finally, I will show how it was operationalized over the past decade by examining research and instructional parameters of interest to the organization and relating them to activities within RCDPM. Since I need to limit the length of this paper, my presentation here must necessarily be an abbreviated one.

Background

Specific work in mathematical diagnosis did not receive much attention until the 1920s. Heddens (1984) reports that the earliest work he found related to error analysis by Myers (1925) and Morton (1925). Both of these individuals studied student errors by looking at student worksheets. About the same time, Buswell (1926) reported his clinical investigations in mathematics which also dealt with the concepts of diagnosis and prescription.

The efforts in the 'twenties to conceptualize diagnosis as both group and individual processes was supplemented by the work of Brueckner (1935) who presented the notion that diagnosis is something which must encompass more than content. He proposed five different areas which should be examined in diagnosis: physiological factors, mental capacity, pedagogical factors, social and environmental factors, and emotional factors.

World War II to 1960

Out of the worldwide experiences of preparedness for fighting World War II, the first great technological war, and the changes coming about in the late '30s, the goals of mathematics began to change. Two major changes in this period were new emphases on diagnosis of problem solving, and diagnosis of understanding.

Process models using pictures and manipulatives began to appear in instruction and in discussions of teaching mathematics during the '30s. In 1938, Morton presented a strong case for looking at these models as a part of the meaningful arithmetic program. It was only a few years later that Spitzer (1948) and Morton (1953) began to talk about these types of models of understanding as important outcomes of the arithmetic program. As a logical outcome of

earlier work on the factors related to mathematics learning, Brueckner and Bond (1955) discussed the importance of teaching and evaluating problem solving performance. This was a shift away from computation as the only major outcome of the arithmetic program.

1960 to Present

Heddens (1984) reports that the '60s were barren years; He was aware of no articles or books devoted to diagnostic and prescriptive work in mathematics. However, with the beginning of the decade of the '70s there were three major works that emerged. These works were by Glennon and Wilson (1972), Reisman (1972), and Underhill (1972). All these works placed strong emphases on the developmental and cognitive relationships between psychological theories and learning mathematics. All of them also emphasized the importance of concept and skill sequences in the diagnostic/prescriptive process. Glennon and Wilson and Reisman used Bloom's (1964) taxonomy, and Glennon and Wilson and Underhill used Gagne's (1970) work to establish content taxonomies. In addition, Underhill used Bruner's (1966) concepts of enactive, iconic, and symbolic knowing as the basis for diagnosing mathematics learning with concrete, semi-concrete, and abstract items. Reisman also included in

her work diagnosis of the affective domain. The main emphasis of Glennon and Wilson and Reisman was in clinical diagnosis; Underhill's emphasis was in group diagnosis. That same year, Ashlock (1972) also continued the earlier work and interest in error analysis with publication of his Error Patterns in Computation.

In the early '70s, RCDPM came into existence because of the vision of one man: Jim Heddens. Jim had the foresight and the resources for convening a group of people for the purpose of fostering long-term focused efforts and progress in the area of diagnostic and prescriptive mathematics. The Council exists because of his vision and the fortuitous timing of available resources.

The RCDPM Mission

> The Research Council for Diagnostic and Prescriptive Mathematics seeks to stimulate, generate, coordinate, and disseminate research efforts designed to understand and/or overcome factors that inhibit maximal mathematics learning.

The mission statement indicates that research is the primary reason for the existence of the Council. I would like to indicate the ways which, over the past decade, have emerged as vehicles for dealing with diagnostic/prescriptive research through the Council. These might be categorized under conferences, publications, and other.

Conferences

The members of the Council have used oral presentations as one major way to contribute to the field. Speeches vary in their orientation from needed research, to reviews and syntheses of literature, reports of clinical research, reports of traditional empirical research studies, and applications of research in classroom and clinic settings. The Wilson Memorial Lecture was established in 1980 following the death of a special colleague, John Wilson, and has been used to bring in specialists in neurology, special education, cognitive psychology, and so on to relate their respective fields to mathematical diagnosis and prescription.

Special research interest groups are evolving. They were first proposed three years ago and the Executive Committee facilitated the development of research presessions at the 1983 conference. One of the presessions was on the teaching experiment methodology and the other one was on the development of common interest research working groups. Following that conference there was a survey made of all members of the Council. From this survey four research working groups were formed at the 1984 conference. These working groups are communicating by mail and plan to have special sessions on the program of the annual meeting.

This will help their deliberations on conceptual and methodological issues.

Publications

Since its inception in 1973, the Research Council has sought to publish annual monographs containing papers on research and other topics related to mathematical diagnosis and prescription. Commencing with the publication of the 1983 monograph, procedures for blind reviews were implemented. In addition to monographs, certain ancillary publications have been printed by the Research Council including bibliographies, annotated bibliographies, dictionaries of diagnostic/prescriptive terms, and Wilson Memorial Lectures. By action of the RCDPM Executive Committee, nearly all RCDPM publications will become available to a much wider audience through the ERIC system in the United States. This is a microfiche system, operated under federal funding, through which copies of thousands of documents are made available on the campuses of all major universities in the country and in many public libraries.

Since the Council was incorporated, it has published a quarterly newsletter. This newsletter contains up-to-date information about activities of the Executive Committee, information about recent publications, messages from the officers, and so on.

By action of the membership at the 1984 annual meeting in San Francisco, RCDPM has formally affiliated itself and has editorial control of the journal, Focus On Learning Problems in Mathematics. Mahesh Sharma of Framingham, Massachusetts, is the owner of this journal and serves as the Chairman of the Editorial Board. Ed Uprichard of the University of South Florida is the first RCDPM Editor. He was appointed for a two-year term. Ed is now inviting manuscripts for the FOCUS review process.

Other

One project underway at this time is a Presidential Commission, chaired by Skip Fennell of Western Maryland College, to develop an RCDPM position paper on Diagnostic/Prescriptive Research Goals for the Year 2000. This Commission will present its report at the 1985 RCDPM conference in Austin, Texas.

President-Elect Leroy Callahan is conducting a survey of RCDPM members to determine their sense of the organization, their interest in participating, their thoughts about publications and conferences, and so on. The results of this survey will be used as part of the continuing work of the Executive Committee to help forge new directions and new projects for our organization.

Realization of the Mission

At the 1984 annual meeting of RCDPM in San Francisco, former Council president Jon Engelhardt made the following statement in his presentation to the membership about research issues:

> RCDPM is one of only a few organization that has placed up front -- in its name -- the fact that it is concerned and committed to research. But unlike most of the others, RCDPM is not exclusively a research organization; it is full-range professional organization (dedicated to promoting maximal mathematics learning) that prominently places research in its professional agenda.
>
> Specifically, it acknowledges that (1) research is critical to increase learning and instruction in mathematics, and (2) if improved mathematics learning and teaching are to result, (a) all types of research must be encouraged and (b) the producers and potential consumers of research must be brought together in a forum that promotes mutual respect, communication and partnership. This is certainly not to downplay the mutual stimulation created by researchers getting together separately, but to "up play" the importance and role of field practitioners in the advancement of knowledge.

With this background, Engelhardt identified five research issues important to the Research Council for Diagnostic and Prescriptive Mathematics:

1. How to best stimulate, coordinate and communicate research activity.

2. How best for research to be incorporated into educational practice.

3. What the nature and areas of research are that should be addressed and promoted within RCDPM.

4. How to focus research efforts to systematically examine a particular area.

5. What role "arithmetic" should play in the research efforts of RCDPM.

With this background, I would like to present data which show how the Research Council has operationally defined its mission through its monographs and conferences. In doing so, I would like to impose a structure on it to look at certain strengths and weaknesses, heavy emphases, and gaps.

To examine the conferences and monographs of the Research Council, I commenced to make an analysis based on research, needed research, how to, and so on. Then I made a second dimension related to the topics of the presentations. Some presentations and papers were tallied more than one time if they dealt in very substantial ways with more than a single cell in the grid. After completing this, I sought to organize them. They seemed to fall rather naturally into four categories. I referred to them as general issues and issues about learners, instruction, and mathematical content. These are similar to some of the categories cited by Ed Begle in his last work, Critical Variables in

Mathematics Education, published posthumously in 1979 in which he sought to document the status of research in mathematics education. In Figures 1, 2, and 3, these four major categories are in the left column. Across the top are five categories on a different dimension: Needed Research, Review/Analysis/Synthesis/ Conceptual, Clinical Research Reports, Group Research Reports, and Documented "How To."

Figure 1 is read as follows. Fourteen of the 31 papers in the first two monographs, or 45% of them, dealt with general issues related to diagnosis and prescription. And at the bottom, 13%, or 4 out of 31 major papers at the first two conferences, dealt with needed research in the area of diagnostic and prescriptive mathematics. There is a wealth of information in Figures 1 and 2 which is of varying interest to specific individuals. It is summarized by category in Figure 4.

Focusing first on topics, you can see that in the first two monographs the two major topic areas were General Issues in diagnosis and prescription, 45%, and Instruction, 36%. In the two most recent monographs, the General Issues decreased to 27%; papers about Learners increased to 35%, and Mathematics Content is now a major topic in 73% of the papers. These represent substantial decreases over the decade in the major areas General Issues and Instruction,

and major increases in topics related to Learners and Content. If you study the monographs, it becomes quickly apparent that in the early years of the Council there was much concern appropriately addressed toward the development of conceptual models, the role of clinics, and test design. These were heavily oriented toward diagnostic and prescriptive processes. Once many conceptual papers were developed in the early years, there was a move in the direction of papers which dealt with specific content. The three content areas which dominate that work are Arithmetic Skills, Concept Learning, and Problem Solving.

In Figure 4, you can also see the orientation of the monographs. The most dominant orientation in the first two monographs was the review/conceptual type paper; it was the orientation of 68% of the papers. A decade later, the efforts are much more broadly dispersed. Forty-two percent of the papers are still dealing with review/conceptual issues, but 27% now report clinical research and 62% group or traditional empirical research. Documented How To, which report developmental projects and models for classroom practice, clinics, and teacher education, is a major orientation in 20% of the recent papers. The global change in orientation is very strongly toward research reporting, constituting 73% of the papers in the two recent monographs.

An additional 17% were research-into-practice articles or conceptual models for implemention of diagnostic/ prescriptive procedures.

Unfortunately, I have no copies of the programs of our early conferences. In the first conference, there were major papers and reaction papers. A small group of mathematics educators sat around a table discussing scholarly issues, and teachers sat in a gallery arrangement. The second conference had a day for talking about research among researchers and a day of talking about models and research results with practitioners. The first two conferences established an organizational orientation toward diagnostic and prescriptive research in the broad sense of the word: doing research, developing conceptual models, and translating research into practice.

In Figure 3, I have placed the contents of recent presentations into the same grid. I did this on the basis of each title and sentence of explanation in the conference program. Once again, some presentations were coded more than one time if they had dual emphases. At the right side of Figure 3, you can see that 19% of the 122 presentations at the 1983 and 1984 conferences dealt with general issues in diagnostic and prescriptive mathematics. Forty-three percent of the presentations dealt with topics related to

learners, 30% of the presentations with instruction in the areas noted, and 38% with specific mathematics content.

Looking again at the orientations of the diagnostic and prescriptive topics, you see that research issues and research needs were a major portion or 20% of the presentations; literature reviews, analyses and conceptual treatments were part of about 18% of the presentations; 10% were clinical research reports and 25% were group or empirical research reports, thus making research reports about 35%, total; 37% were curriculum and activity implementations, referred to as "How To" presentations, and 21% dealt with models in diagnostic and prescriptive development projects.

It is clear that the members of the Council who contribute to monographs and conferences are very diversified in their interests and scholarship efforts. In my analysis I tried to distinguish between arithmetic concepts and arithmetic skills. Arithmetic skills in general are those algorithmic skills related to whole number and rational number computation. I personally do not think that collapsing arithmetic concepts into that rubric is an adequate way of dealing with the issue because the development of concepts, whether arithmetic, geometric, measurement, or whatever, is an essential area of study and

will continue to be important even if algorithms tend to decline in importance. Based on these data, it does not appear to be a correct statement that the Research Council spends too much effort on arithmetic skills.

What makes the Council unique? Is it orientations? No. Is it topics? No. It is a diagnostic/prescriptive focus. Council members are interested in all the areas cited in the grid. What makes their work cohesive is the slant they take. Each discussion of needed research, each clinical or empirical investigation, each implementation model is characterized by a focus, a lens if you will, which seeks to extend our understanding of all the topics as they relate to diagnosis and prescription. In those areas where Council Members are not cited, there is already work being done so we must seek to bring scholars into our domain of activities or to make special efforts to relate current activities in these domains to the diagnostic/prescriptive focus of RCDPM.

People who are interested in children's mathematical learning need to maintain constant and on-going dialogue. Teachers need to talk with teachers. Teachers need to talk with researchers. And researchers need to talk with researchers. The Research Council is constantly looking for ways to make all three of these communication patterns

operate. If researchers only talk with researchers, and teachers only talk with teachers, change will not occur in the mathematical instruction of the world's schools. As past-president Bob Ashlock stated at the 1984 annual meeting,

> One of our unique characteristics as an organization of math educators with a special interest in those who are not learning as well as they might, is that we have classroom teachers as members, as well as researchers and teacher educators.... We believe that practitioners are essential to the organization if we are to be the community of professional people who are concerned with those who are having difficulty learning mathematics. Individuals in the various professional roles need to be represented,... and they need to speak out....

All of us share a deep and abiding concern for the steady progress of children in developing, learning, and using mathematical concepts and skills. We can look at where we are and say that much has been accomplished. We must look ahead of us and see that there is still much to do.

Figure 1. FIRST TWO MONOGRAPHS -- 31 Papers*

	Needed Research	Review/Analysis/ Synthesis/Conceptual	Clinical Research	Group Research	Documented "How To"	TOTALS
GENERAL						
General Research Issues		2				2
Clinics	2	2				4
Diag/Remed Term.		1				1
Test design		3	1	2		6
Hierarchies		1				1
						14 (45%)
LEARNERS						
Sp. Ed/Gifted/Brain		2				2
Ability/Aptitude						0
Affective domain		1				1
						3 (10%)
INSTRUCTION						
Diag. Processes	1	6			1	8
Remed. Processes		1				1
Tchr Educ	1	1				2
Technology						0
						11 (36%)
CONTENT						
Arith. Skills			1			1
Concept Learning		1	1			2
Problem Solving						0
Vocabulary/Rdg						0
Estimation						0
						3 (10%)
Totals	4 (13%)	21 (68%)	3 (10%)	2 (6%)	1 (3%)	

* Does not include reaction papers.

Figure 2. TWO MOST RECENT MONOGRAPHS -- 26 Papers

	Needed Research	Review/Analysis/ Synthesis/Conceptual	Clinical Research	Group Research	Documented "How To"	TOTALS
GENERAL						
General Research Issues						0
Clinics		1		1		2
Diag/Remed Term.						0
Test design				4		4
Hierarchies				1		1
						7 (27%)
LEARNERS						
Sp. Ed/Gifted/Brain		2		1	1	4
Ability/Aptitude		2				2
Affective domain				2	1	3
						9 (35%)
INSTRUCTION						
Diag. Processes						0
Remed. Processes						0
Tchr Educ						0
Technology		3	1			4
						4 (15%)
CONTENT						
Arith. Skills		1	2	3	1	7
Concept Learning		2	3	1		6
Problem Solving			1	2	1	4
Vocabulary/Rdg			1			1
Estimation					1	1
						19 (73%)
Totals	0 (0%)	11 (42%)	7 (27%)	16 (62%)	5 (20%)	

Figure 3. Conference Presentations, 1983 and 1984 (N=122)

	Issues/ Needs/	Reviews/ Conceptual	Research: Clinical	Research: Group	Curriculum and activities How to	Models/ Development/ Projects		
GENERAL ISSUES								
Methodology	1	2	1		2		6	
Writing & Reviewing					1		1	
Math learning		1					1	
Clinics	1	1					2	
Research Working Groups	4						4	
Specific tests	1	1			1	6	9	
							23	(19%)
LEARNERS								
Epistemology		1					1	
Prescript/Remediation	1	2	1		4	1	9	
Sex Equity/Differences		1		1			2	
Sp Ed/Brain/Gifted		1		3	5	2	11	
Dev. Psyc				1	2	1	4	
Adult Lrnrs	1						1	
Affective	2	2	1	6	3	1	14	
Abilities	1	1					2	
Failure				1		1	2	
Reasoning/Logic						1	1	
Lrng Styles/Modalities	2	2			2		6	
							53	(43%)
INSTRUCTION								
Tchr Ed	1			2	2	2	7	
Diag tching			1				1	
Technology	5	3	2	1	4	8	23	
Parents					1		1	
Group Diag.	1				1		2	
Clinical Diag.					1		1	
Prescription					1		1	
Manipulatives			1				1	
							37	(30%)
CONTENT								
Algebra	1				1	1	3	
Concepts	2	1	3	5	5	1	17	
Calculus				1			1	
Arithmetic (comp)			2	3	3		8	
Geometry			1	1			2	
Rdg/Lang/Vocab		3		2	1		6	
Problem Solving	1			3	5	1	9	
							46	(38%)
	25 (20%)	22 (18%)	13 (10%)	30 (25%)	45 (37%)	26 (21%)		

Figure 4. COMPARISON OF MONOGRAPHS

	First Two	Two Most Recent	Change
Subject			
General Issues	*(14) 45%	*(7) 27%	(-7) -18%
Learners	(3) 10%	*(9) 35%	**(+6) +25%
Instruction	*(11) 36%	(4) 15%	##(-7) -21%
Content	(3) 10%	*(19) 73%	**(+16) +63%
Orientation			
Needed Res.	(4) 13%	(0) 0%	(-4) -13%
Review/Conceptual	*(21) 68%	*(11) 42%	##(-10) -26%
Clinical Research	(3) 10%	*(7) 27%	(+3) +17%
Group Research	(2) 6%	*(16) 62%	**(+14) +56%
Documented "How To"	(1) 3%	*(5) 20%	(+4) +17%

*≥20%

**Increase≥20%

##Decrease≥20%

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THE INTERACTION OF KNOWLEDGE AND COGNITIVE PROCESSES IN DIAGNOSIS AND PRESCRIPTION - IMPICATIONS FROM ARTIFICIAL INTELLIGENCE RESEARCH

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INTRODUCTION

> "In carrying out instruction, a student's strengths and weaknesses in learning in a particular domain could be assessed. If a student has acquired much of the specific knowledge needed for subject matter mastery, instruction aimed primarily at general self regulatory skills might be indicated. However, if a student shows competence in general problem solving and self-regularity strategies, and is likely to employ them to guide learning in a new area, then an emphasis on knowledge and skill specific to a domain is called for."
>
> Glaser (1980), discussing Brown (in press)

Early work in artificial intelligence research was in knowledge lean domains such as tower of Hanoi puzzles (see Figure 1 below) and the implications of research findings related to general cognitive processes. Later research has been in knowledge rich domains such as radiography, physics or mathematics. These, as Glaser (1980) excellently summarizes, have refocused attention on knowledge structure

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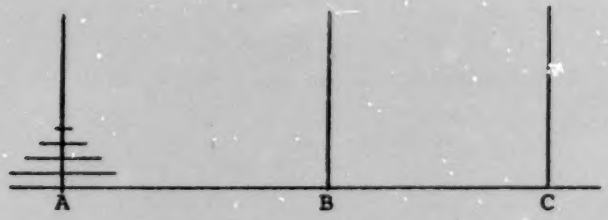
giving rise to a dilemma that forces us, in Glaser's terms, to consider teaching thinking in terms of "knowledge structure-process interactions". In this paper we wish to explore how the interrelationship between domain specific knowledge and general cognitive processes, or heuristics, can be used in diagnosis and remediation - basing this discussion on artificial intelligence research, particularly our own efforts to cumulate this research into a general model of the human learner as an information processing system.

Figure 1: The tower of Hanoi puzzle

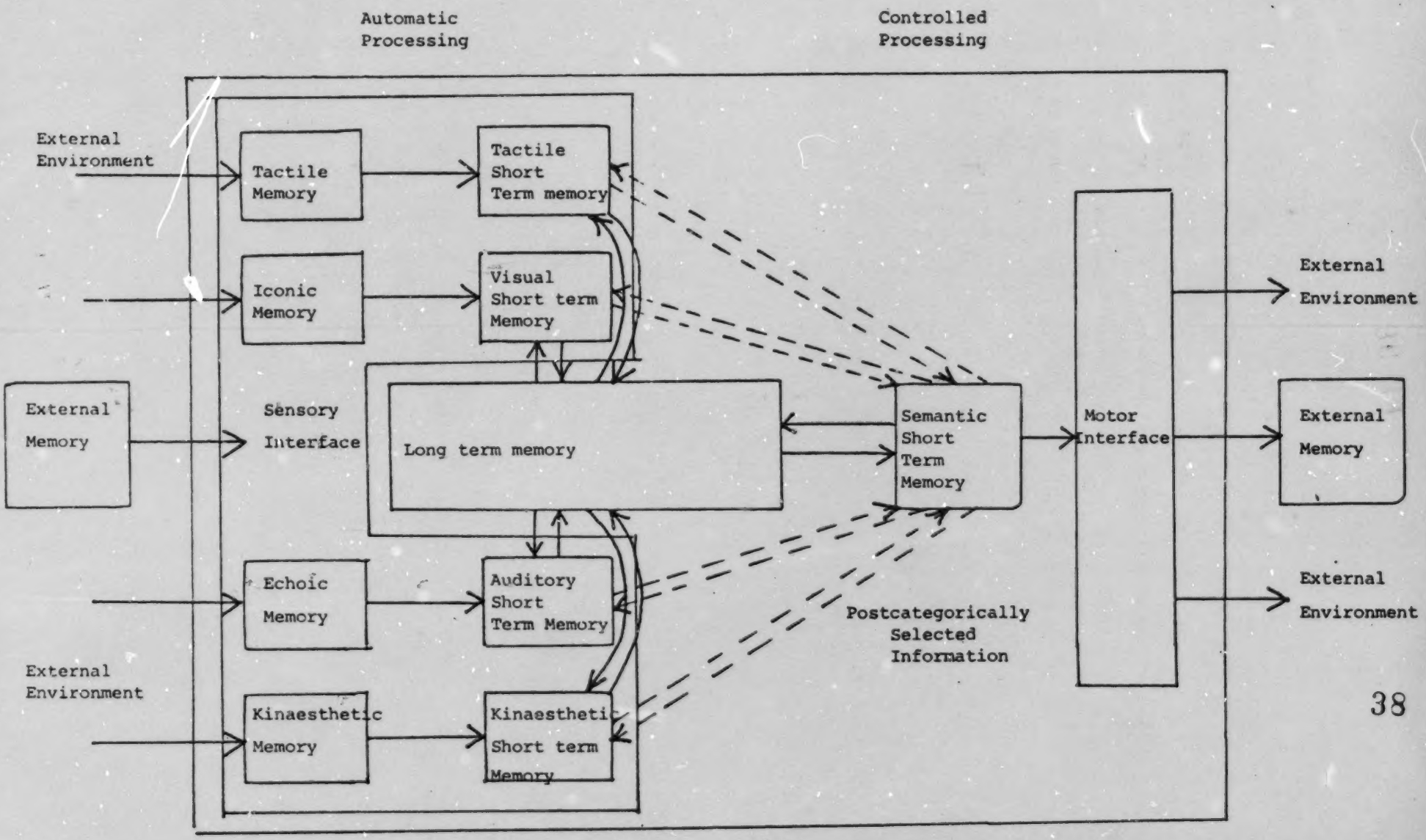
Move the discs from A to C, using B, such that each move is of one disc and a larger disc is never placed on top of a smaller.



THE HUMAN LEARNER AS AN INFORMATION PROCESSING SYSTEM

In Nason and Cooper (1984), we have developed an information processing model of the human learner (see Figure 2 below), in which we have endeavoured to incorporate the latest findings of artificial intelligence research, particularly those of Anderson (1976, 1980 and 1982), Glaser (1984), Klahr and Wallace (1976), Newell and Simon (1972), Van der Heijden (1981), Wallace (1982) and Wallace, Klahr and Bluff (in preparation).

Figure 2: A model of the human learner as an information processing system (Nason and Cooper, 1984).



The model is based on a constructivist epistimology (Hunting, 1980; Von Glaserfield, 1978). Knowledge and learning are evolutionary in nature and learners are viewed as goal directed organisms that structure (anew in each learner) their own knowledge, a knowledge which is best understood as internal cognitions. The model, and the assumptions underlying it, are completely described in Nason and Cooper (1984). For this paper we will summarize the major parts, with particular emphasis on aspects important to diagnosis.

The architecture of the model

As illustrated in Figure 2 above, the model has five major components:

i) Long Term Memory;
ii) Semantic Short Term Memory;
iii) External Memory;
iv) Motor Interfaces; and
v) Sensory Interfaces.

(1) Long Term Memory (hence referred to as LTM)

The LTM is the store which holds the Human Information Processing System's entire repertoire of concepts, processes and skills. It is therefore the final repository of all information that has been processed and relatively permanently encoded by the system. The LTM also contains an episodic memory which is the sequential record of the system's processing activities.

A number of characteristics clearly differentiate LTM from other stores within the system. Included in these are the following:

(i) relatively unlimited capacity;

(ii) a relatively low forgetting rate; and

(iii) a relatively slow retrieval rate.

(2) Semantic Short Term Memory (hence referred to as SSTM)

The SSTM is the focus of controlled or conscious (Schneider & Shiffin, 1977; Shiffin & Schneider, 1977) processing of information within the information processing system. It is therefore useful to think of the SSTM as a "working memory" that holds knowledge currently in use for a strictly limited amount of time, unless this knowledge has undergone rehearsal. The most important function of SSTM is that of active control of thinking, problem solving and general memory processes (Shiffin, 1976; Klahr & Wallace, 1976; Anderson, 1980).

Two of the most important memory processes which occur in SSTM are rehearsal (a process whose primary function is to maintain information traces in SSTM) and coding (a process whose primary function is to facilitate the effective storage of information in LTM).

A most important characteristic of the SSTM, which differentiates it from other stores within the system, is that it has strictly limited capacity.

(3) External Memory (hence referred to as EM)

The EM is an augmentation of the SSTM external to the system. It acts as an external store where information can be temporarily set aside from the SSTM and from which it can be quickly retrieved by the SSTM when it is required. The EM increases the capacity of the SSTM considerably and thus enhances the stability of memory. Examples of EM include using pen and paper, flow charts, diagrams, pictures and concrete models.

(4) Motor Interface (hence referred to as MI)

The Motor Interface is the component of the system through which output to the external environment is directed. This out put can take the form of speech, muscular movements, physical actions (both simple and complex), writing, drawing and gestures, etc.

(5) Sensory Interface (hence referred to as SI)

Within the Sensory Interface of this model, there are four parallel sensory channels (visual, auditory, tactile and kinaesthetic) of theoretically unlimited capacity (Sperling, 1960; Seamon, 1984; Van der Heijden, 1981) and two sets of memory buffers (primary and secondary).

The Primary Buffers are very short term sensory stores whose function is to temporarily hold sensory information until encoding can begin. In effect, they make us capable of processing information that is no longer externally available (Seamon, 1981). In this model, each sensory channel has one primary buffer: Iconic Memory in the Visual Channel; Echoic Memory in the Auditory Channel; Tactile Memory in the Tactile Channel and Kinaesthetic Memory in the Kinaesthetic Channel.

The Secondary Buffers form a distinguishable but not separate substage in the stream of sensory information processing through the various sensory channels (Orik & Lockhart, 1972; Shiffin & Schneider, 1977). This model therefore has four secondary buffers: Visual STM (hence called VSTM) in the Visual Channel; Auditory STM

(hence called ASTM) in the Auditory Channel; Tactile STM (hence called TSTM) in the Tactile Channel and Kinaesthetic STM (hence called KSTM) in the Kinaesthetic Channel.

Within this model, the Secondary Buffers have two major functions. Their first major function is to encode the incoming sensory information. Encoding may be defined as the process of interpreting a stimulus and storing a representation of that interpretation in the SSTM, translating the incoming information from a sensory form into a semantic form.

Their second major function is as memory stores where sensory information is associated with other information within the same channel and/or where the information is restructured within its sensory form. For example, it is envisaged that the VSTM is the memory store within the human IPS where visual imagery is used to restructure and facilitate the solving of many types of mathematical problems. VSTM thus may in certain contexts act as an adjunct to SSTM because it can be used as a place where certain information may be temporarily set aside from SSTM for further encoding and restructuring.

In effect, this may be an additional way for the Human IPS to overcome the limited capacity of SSTM. VSTM (and probably the other secondary buffers too),therefore, has an additional function which is similar to that of EM.

Representation of knowledge

Knowledge is represented in the system by a single network of nodes at which are stored production systems (Newell and Simon, 1972). Each node has three components:

(1) a definition list - a sequence of processes, stored as production rules (Newell & Simon, 1972) which define what the node represents;

(2) an experience list - information about states and transformations, which have been consistently detected in previous processing of the node and which enable future processing to detect novelty; and

(3) a description list - information about the node's connectivity with other nodes (these are the nodes to which activation may spread when the original node is being processed).

Figure 3: Types of Nodes

Within the network of knowledge stored in LTM, it is possible to identify many different categories of nodes. These different categories of nodes can be hierarchically organized on a three dimensional grid such as Figure 3 below.

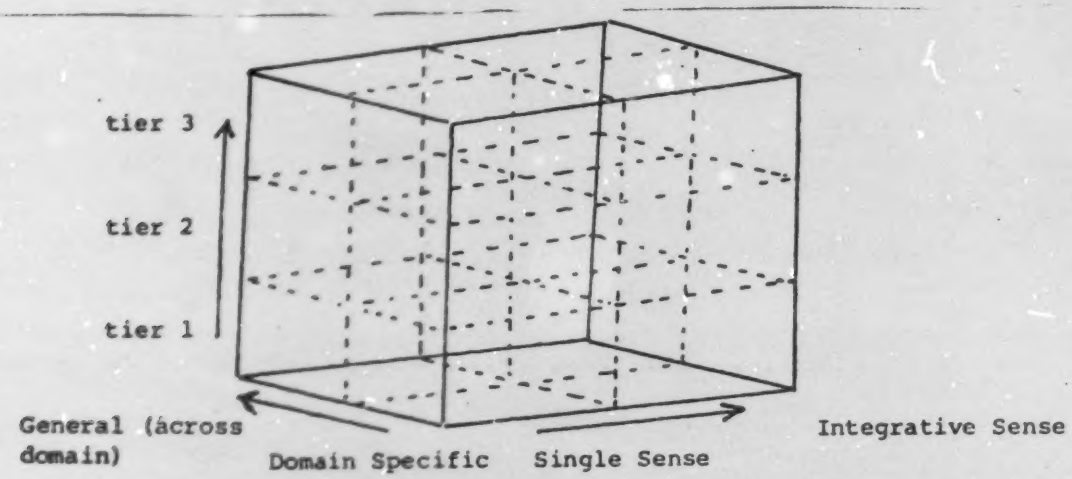


Figure 3

The division of nodes into three vertical tiers is a notion taken from Klahr and Wallace (1976). The simplest and most specific nodes within the network are to be found in Tier 1. The more complex and general strategic nodes are to be found in Tier 2. The Tier 3 nodes contain the systemic productions which are the superordinate executors of the IPS. These productions are for example involved in the processes of detecting consistent sequences and rule generation. They thus have a crucial role to play in the self-modification procedures of the system.

With TIER 2, there are at least two sub-categories of strategic nodes: those nodes which Klahr & Wallace refer to as "housekeeping" nodes (or Glaser refers to as a self-regulatory) whose function is to govern the setting and the general administration of goals within the system and problem-solving strategic nodes which come into action when an impasse is reached with the application of Tier 1 nodes in problem-solving situations. The Tier 2 nodes thus operate both on other Tier 2 nodes and on Tier 1 nodes to facilitate the compilation of more complex and general nodes within LTM.

The notion of categorizing the nodes along a continuum from Domain Specific to General (across domains) has been developed as a result of recent research into problem solving discussed earlier, in which it has been found that the development of general (across-domains) problem solving strategies is a much more complex task than it was generally thought to be (see Glaser, 1984).

The further categorization of nodes along a Single Sense to Integrative Sense continuum was the result of the necessity. For this model to adequately explain the functioning of the Sensory Buffers, it was essential to locate nodes concerned with the perceptual processing of sensory input

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within the LTM. Much of this perceptual processing involves only one sensory modality. However, as some perceptual processing by the human IPS involves the integration of information acquired from more than one sensory modality, it was necessary to extend this dimension along a continuum to include those nodes which require sensory input from a number of modalities in order to be activated.

The functioning of the system

The functioning of the model centres around the activation of one or more nodes stored in LTM, an activation fired by information coming from outside the system (via the memory buffers) or inside the system (by spread of activation from other nodes) or both.

Most of the sensory stimuli entering the sensory interface undergoes some primitive initial encoding by the system. As was explained previously, almost all of initial encoding within the sensory buffers is done unconsciously and automatically.

Although all of this primitively encoded sensory information proceeds further into the system, most of it does not undergo conscious, deliberate processing. This is mainly due to the limited capacity of SSTM. Which information undergoes conscious processing depends on the weighting each chunk of information has acquired. Only some information satisfies the presend "needs" of the system sufficiently (in terms of e.g. motivation, familiarity, recency, interest) to be postcategorically selected (Van der Heijden 1981) for conscious processing in the SSTM.

Within SSTM, there are two levels of conscious processing: focal and peripheral.

Focal conscious processing is limited to a single active node in the network during each processing cycle (Wallace, 1982). The node currently in focal consciousness takes up most of SSTM's limited capacity. A major consequence of this is that it is the information produced by this node which primarily determines the input available via SSTM to other nodes. This node therefore has a great influence in determining subsequent node activation.

If sufficient of the SSTM's limited capacity is free, a second node may operate successfully in peripheral consciousness. The information produced by the "firing" of productions in the peripheral node also enters SSTM and it too may influence subsequent node activation.

The peripheral level of conscious processing is a most important aspect of this model's explanation of the problem solving process. The system involved in the process of problem solving has to search beyond the arena of focal consciousness in order to locate the information necessary for the solution of the problem.

As was stated earlier, most information entering the system is not consciously processed. Much of this information decays and is lost forever. However, some information not consciously processed does get stored in LTM and often forms an important part of the system's repertoire of knowledge. In order to account for this phenomena, this model allows for nodes other than those occupying focal and peripheral consciousness to be active during each cycle. However, it should be noted that information produced by

these nodes is not placed in SSTM and it therefore is not generally available as input to other nodes. According to Wallace (1982), the influence of this information on subsequent node activation is confined to spread of activation by direct transmission to the working spaces of other nodes.

The process of acquiring new knowledge

The process of acquiring new knowledge takes place through the detection of regularities in the system's interactions with the environment. The system scans the episodic memory and if elements match to a level required by the system, a new node is created, stored in LTM and defined in terms of the production lists which were responsible for the consistent sequence in the episodic memory.

One of the major principles which governs the functioning of the system is the elimination of redundant, inefficient processing from the system. Tier 3 systemic nodes and Tier 2 "housekeeping" nodes constantly scan sequences stored in episodic memory in order to detect this sort of processing.

Modifications to the structure of the nodes, and in some cases to the to the structure of the network of nodes within LTM, are then made as a result of attempts by these higher order Tier 2 and Tier 3 nodes to eliminate the redundant processing from the system.

The system achieves this improvement to its information processing capabilities by a number of compilation procedures.

First, there is the procedure which Anderson (1980, 1982) refers to as composition. According to Anderson (1982), composition takes sequences

of productions that follow each other in solving a particular problem and collapses them into a single production that has the effect of the sequence. This produces a considerable improvement in speed by creating new operators that embody the sequences of steps that are used in a particular problem domain.

Second, there is proceduralization (Anderson, 1982) which builds versions of the productions that no longer require the domain-specific information to be retrieved into SSTM and/or the sensory buffers in order that they may "fire". Proceduralization thus reduces the number of chunks of information in the condition part of the production by removing the redundant ones and leaving only those conditions which experience shows usually results in producing the required action.

Modifications to the LTM produced by composition and/or proceduralization are provisional until the system confirms that the economy in operation has been achieved without impairment to the system's interaction with the environment (particularly in terms of the perceived correctness of the actions produced by the new productions and/or production lists). This procedure of confirmation is the system's defence against the problem of overgeneralization. This defence, however, is not fail-safe because it relies on the perceived correctness of the actions.

One major result of this procedure of confirmation is that the system realizes the need to restrict the range of application of overly general productions. That is, it identifies those conditions which indicate when the generalized production is not appropriate. This restriction of the range of application of a production is referred to as the process of discrimination (Anderson, 1982).

IMPLICATIONS OF THE MODEL

If you believe, as we do, that mathematics is a way of thinking and that improving mathematical ability is improving thinking in the domain of mathematics (and out of the domain of mathematics), then it seems evident from the model that instruction should aim to develop a system in which functioning in the following three areas is enhanced:

1. Acquisition of mathematics knowledge.

The ability of the system to acquire knowledge is obviously of fundamental importance as it is the prime focus of instruction.

Within the model espoused in this paper, instruction does not supply this knowledge but supplies the environment in which the system can structure its own knowledge. Hence instruction should focus on the following three functions:

(i) The detection of regularity.

Regularities are detected as the system scans episodic memory, trying to match up the elements stored there to see if they can be integrated to form a new node. The degree of matching required depends on the level of agreement between the elements which the system requires before they are regarded as a match (the Matching Index Criterion of the system or MIC), When, as a result of a number of scans, the quantity and quality of the matches reaches a critical point (the Critical consistency level or CCL), a new node encapsulating the detected consistency is added to the LTM.

How learners acquire new knowledge is affected by their ability to detect regularity. Some children require many examples of very similar matches in episodic memory before they can create a new node. Others, require only a

few partial matches to create a node. Within our model this detection is also affected by the quality of nodes' experience lists. Instruction should try to match the number and type of experiences given to learners' MIC and CCL levels. In particular, many "slow learners" require many very similar examples before regularity is detected.

(ii) Composition, Proceduralization and Confirmation

Instruction should provide opportunities for students to compose and proceduralise knowledge to form higher level nodes which allow the system to operate more economically. This has to be followed by experiences which allow learners to confirm their generalizations and so detect and modify over generalizations. Teachers have to focus on similarity and difference, on prediction and checking, to achieve these multiple objectives.

(iii) The acquisition of tier 2 and 3 nodes

Thinking involves the actions of all types of nodes. Learners have to have a knowledge base of tier 1 nodes on which to operate, but many of the mathematical requirements of modern society (e.g. problem solving re-education, transfer) depend on the higher intellectual abilities encompassed in strategic and executor nodes.

2. Strategies to overcome SSTM capacity limitations.

Many of the problems and activities required in a modern mathematics curriculum have a SSTM load that puts at risk (and in some cases, prevents) the ability of the learner to consciously process the information. Instruction should focus on techniques to overcome this limitation in SSTM. This is particularly so for poor thinkers who may have very low SSTM capacity.

These techniques can be usefully categorised into these three types:

(i) External Memory Strategies

Drawings and doodlings can allow the SSTM, with its limited capacity, to handle complex situations. The SSTM can interact with the EM much more efficiently than it can with the LTM. Instruction should focus on promoting the use of EM strategies such as the use of pen and paper drawings, flowcharts and doodlings.

(ii) Secondary Memory Buffer Strategies

The conscious and unconscious processing that occurs in these buffers gives opportunity for the system to overcome the capacity limitations of the SSTM and to undertake processing in parallel. There seems some evidence for example that processing important to problem solving occurs from the Visual short term memory. Alongside this processing has to be the development of nodes which scan the VSTM.

(iii) Rehearsal Strategies

Rehearsal strategies prevent important information from slipping out of the SSTM while other things are being done.

3. Selection for conscious processing .

Many learners who have a poor mastery of mathematics seem, in many instances, unable to find the appropriate information to process and, in other instances, seem unable to match the appropriate information with the required processing. They are unable even to start the problem.

What is selected for processing is determined by motivation, interest, familiarity and recency. The focal node is very important in this selection, as is the structure and organisation of existing knowledge in LTM.

A highly schematised knowledge (Glaser, 1984) allows for appropriate alternatives to be trialled in thinking through good use of peripheral processing and a wide and appropriate spread of activation. Such knowledge has extensive description lists leading to related nodes which may be more appropriate for the information being processed.

Instruction to improve the match of processing and selection should focus on three sections of the system:

(i) Definition lists

The limited capacity of SSTM ensures that only some information proceeds to conscious processing. What information this is, depends on the weighting each chunk of information has acquired in the primitive initial encoding that has taken place in the secondary memory buffers. This weighting is dependent on, amongst other things, the potency of the nodes which were used to encode the information and how well the chunk of information matches the conditions required to "fire" productions in the system's more potent nodes or nodes that have recently been active. All this is dependent on the definition lists these nodes have. Breakdowns in processing at selection occur if the necessary information is unfamiliar and the previous nodes that have just been processed are inadequate.

(ii) Experience and Description lists

Experience and description lists also affect selection for conscious processing. Description lists determine which nodes are activated by spread of activation from previously fired nodes. One of these may be responsible for selection. Experience lists determine what the system recognises as novel and, in terms of the model, what the system recognises as a "feeling of well being" - the "motivation" that is a large determiner of what is selected.

(iii) "Housekeeping" strategies

The limited capacity of the SSTM causes many problems for the system's conscious processing. The SSTM often finds that there are many situations when it is trying to hold too many goals and too many chunks of information to allow for effective processing. In these situations, "housekeeping" (in self-regulatory nodes place some of the goals on hold for later activation and move some of the not so immediately required chunks of information into a situation from where they are continually "rehearsed" by SSTM to keep them available for the later processing. Information in the buffers that relates to these goals on hold or to the rehearsed information (or to nodes that will be activiated by the rehearsed information) has a heavy weighting in the selection process. Rehearsal, in particular, is very important for selection as it prevents important information from slipping out of SSTM while other things are being done.

INTERRELATING KNOWLEDGE STRUCTURE AND COGNITIVE PROCESSES

Errors in thinking arise when nodes are inadequate for the task. In these cases, tier 2 and 3 nodes are activated by the system to 'repair' the deficiency resulting in a new node (see the Repair Theory of Brown and Van Lenh, 1980). If the activation of this new node is not correctly confirmed (negatively), then the over-generalization can become fixed as an internally appropriate but externally incorrect response to the task. The particular problem is of course that such repairs, or generalizations (or in our terms, compositions and proceduralizations), are an important part of the learning process. Problems only arise when they grow to the point that the system cannot use negative confirmation to discriminate and restructure to more appropriate nodes.

The inadequate growth of nodes can be the result of bugs in the system - of breakdowns between and within various components of model in Figure 2. We are not focussing on this aspect of diagnosis and prescription in this

paper and so we will do no more than point to the possibility of this implication of the model. We will continue this paper by looking at the categories of nodes available in the system's representation of knowledge to develop some alternative approaches to diagnosis and remediation.

SOME ALTERNATIVE APPROACHES TO DIAGNOSIS AND REMEDIATION

Classical diagnosis and remediation focusses on tier 1 nodes, the specific nodes of the system. Modern research in artificial intelligence is reinforcing the appropriateness of such a focus even for problem solving. But, as the model explicates, these specific nodes operate in relation to the tier 2 strategic nodes and the tier 3 executor nodes. Hence we have three implications:

(1) Diagnosis needs to look at all three tiers of nodes

It is important, for tier 2 strategic and tier 3 constructor nodes as well as tier 1 nodes, that teachers systematically organise and sequence instruction. In particular the development of tier 2 and 3 nodes can not be left to chance and diagnosis should be used to help match instruction to the capabilities of the learner. This may require new methods of diagnosis, particularly new observation skills. The diagnosis will have to focus on how learners go about their solutions as well as whether they are correct.
It is not our intention to go further into what has to be done here, and why, in this paper. Cooper and Nason (1983) discusses more fully why diagnosis and prescription is the appropriate instructional strategy for tier 2 problem solving strategic nodes. Cooper and English (1984) are developing some instruments for evaluating behaviour generated by the activation of tier 2 strategic nodes.

A very important level which has to be watched is the tier 2 "housekeeping" strategic nodes. In an example studied by Nason (in preparation), a 3-4 year old girl was given the task of sharing lollies fairly between two puppets. This meant that she had two goals - to share and to be fair. She could not simultaneously be both, so "housekeeping" nodes put fair on "hold" while share remained activated. At the same time fair was continuously rehearsed so that it was not forgotten.

(2) Inadequacies in tier 1 nodes can be remediated by instruction in higher level nodes

The tier 2 and 3 nodes act on the episodic memory and the tier 1 nodes to develop new tier 1 nodes. If a learner is deficient in tier 1 nodes, particularly domain specific knowledge, then the system can acquire them by detection of regularity or adaptation, if it has higher level nodes appropriate to the information the system is receiving. That is, general cognitive processes can assist the system to structure domain specific knowledge.

Cooper and Nason (1983) have analysed the problem solving strategies of Meiring (1980) for their effect on information processing and identified which of these strategies could be affective for common weaknesses in mathematical ability (see figure 4 below for some examples).

Figure 4: Strategies (Meiring, 1980) appropriate for common difficulties in Mathematics learning (taken from Cooper and Nason, 1963)

Characteristic	Strategies
Small repertoire of concepts and processes (low achiever).	Look for a pattern/Act it out/Drawing/ identify wanted, needed, given/Looking back strategies/Construct a table/ Guess and check/Solve a simpler problem.
Can only work at iconic and enactive (Bruner, 1966) levels of knowing (problems with symbolic work).	Look for a pattern/Act it out/Make a model/Drawing.
Unstructured and non-related knowledge (rote learnt pigeon-holed facts only).	Look for a pattern/Act it out/Make a model/Drawing/Looking back strategies/ Construct a table/Guess and check/ Work backwards/Solve a simpler problem.
Language difficulties	Act it out/Make a model/Drawing/Select appropriate notation/Relate the problem/ Identify wanted, needed, given/Write an open sentence.
Low self-concept (lacking confidence).	Look for a pattern/Act it out/Make a model/Guess and check.
Limited SSTM capacity.	Act it out/Make a model/Drawing/ Select appropriate notation/Identify wanted, needed, given/Looking back strategies/Account for all possibilities (Systematically)/Guess & check/Solve a simpler problem.

(3) Inadequacies in tier 2 and 3 nodes can be overcome by first developing tier 1 nodes

The process by which higher level general cognitive ability nodes are developed is by modification, or adaptation, of lower level tier 1 nodes by the processes of composition and proceduralization (with subsequent confirmation). Therefore, deficiencies in such nodes could be overcome by setting up appropriate learning of lower order domain specific tier 1 knowledge. In fact, it seems plausible, that general tier 2 problem solving strategies are best structured by learners, in a knowledge rich domain like mathematics, by moving from domain specific tier 1 nodes to domain specific tier 2 nodes and then across to general tier 2 nodes (although this may require a corresponding development of tier 1 general knowledge). As Glaser (1984) states this would require a "Socratic" instructional style,

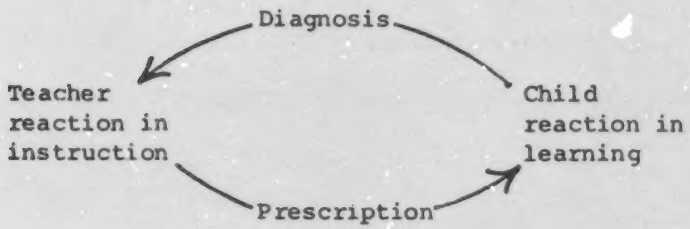
> "teaching specific knowledge domains in interactive, interrogative ways so that general self-regulatory skills are exercised..."

As a particular example, the strategy of "make a drawing, diagram or graph"(Meiring) 1980) may be advantageously taught by extending the learner's consideration of pen and paper algorithms and doodlings that are part of the core arithmetic.

CONCLUSION

Instructing learners who structure their own knowledge requires a diagnostic-prescriptive cycle as in Figure 5 below:

Figure 5: The diagnostic prescriptive cycle



Instructing learners in thinking requires the interaction and balance of general cognitive processes and domain specific knowledge. If used appropriately, instruction in domain specific knowledge will enhance instruction in general cognitive processes and vice versa. Diagnosis and prescription will be the better for taking account of this interaction. Generalization is an important learning technique and overgeneralization is an important part of it. It leads to both learning and error. Instruction therefore, needs to stress similarities and differences and not be too early (or too late) in picking up overgeneralization.

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MASTERING SUBTRACTION FACTS: IMPLICATIONS FROM AMERICAN AND AUSTRALIAN RESEARCH

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INTRODUCTION

Mastery of the 100 basic subtraction facts is a stumbling block for many elementary school students. The fact that subtraction facts are harder for children to learn than addition facts is well documented (e.g. Suydam and Weaver, 1975). The recent results of the Second National Assessment for Educational Progress in Mathematics in the U.S. (Carpenter, et al, 1980) and the national Literacy and Numeracy Investigation of Australian Schools (Keeves, 1977) both highlight the difficulty.

FIRST U.S. STUDY: 1983-84

Carpenter (1983) identified a number of strategies commonly used by children when solving subtraction word problems. Though labelled differently, these same strategies were observed by Thornton (1984) in her study of the way in which children derived answers to unknown subtraction facts. In both studies Counting On ("Counting Up from given" in Carpenter's terminology) proved to be the strategy most frequently used.

Example: 9 - 6. "6--7,8,9. 9 - 6 = 3."

Carpenter found that strategies used by children to solve written word problems generally reflected the semantic structure of the problem when objects were available for use. However, when objects were removed, students often reverted to Counting On.

Table 1 presents a profile of 595 U.S. students who were recently involved in the subtraction study by Thornton (1984). The following criteria were satisfied by all children included in the study:

(1) Concrete manipulation or picture representation to correctly check answers to simple subtraction combinations such as 5 - 2.

(2) Less than 70% accuracy on a two-minute written test containing 54 subtraction facts.

Table 1

Students Stratified by Nature of Instruction and by Grade Level of Mathematics Book Used

(1983-84 U.S. Study)

	Number of Students Interviewed			
	Attending only regular class math	Also receiving math help from Chapter 1 staff	Also receiving math help from LD resource staff	Total n
Grade 1	184	-	5	189
Grade 2	192	-	12	204
Grade 3+	48	47	16	111
Grade 4+	39	38	14	91
				595

+Only those students in a class who scored less than 70% correct on a written, two-minute, 54-item basic subtraction fact test were interviewed.

The purpose of the structured interview and follow-up observations during mathematics instruction was to identify the strategy which was most consistently used by each child to derive answers to unknown subtraction facts. The U.S. investigator and a trained graduate student assisted teachers in carrying out the interviews/observations.

Table 2 illustrates the changing pattern in the use of strategies from grade 1 to grade 4. Some older children maintained the Counting On strategy. Others used Addition to answer unknown facts. A larger percentage employed the strategy of Choice. This strategy involves either Counting On or Counting Back to answer a specific fact, whichever seems faster. In some cases older or more capable students utilized other strategies based on pattern searching or relationships.

Table 2

Dominant Strategy for Answering Unknown Subtraction Facts*
(1983-84 U.S. Study)

		Grade 1	Grade 2	Grade 3	Grade 4	Overall Average
	Number of Students:	189	204	111	91	595
	Month of Interview:	April 1983	October 1984	Feb/Mar 1984	Feb/Mar 1984	
	Strategy					
	1. Modeling (using objects or fingers)	42	40	5	2	22
No objects used	2. Counting On (most always)	36	38	30	28	33
No objects used	3. Choice (counted on or back, whichever seemed faster for a given fact)	6	7	50	52	27
No objects used	4. Addition	1	1	12	15	7
No objects used	5. Counting Back (most always)	14	15	4	2	9

* Rounded to nearest whole percent.

Preference for the Choice strategy by older, lower level students should be made in light of their previous instruction in subtraction. From grade 1 on, textbook series used by all students in the study emphasized addition - subtraction "families". Since this emphasis prefaced practice at every level, it might be assumed that using Addition to find unknown subtraction answers was the goal. It is not known whether students in the study understood the addition-subtraction relationship. It is clear that a large number of them did not use Addition. The very early emphasis in grade 1 on addition-subtraction families, and the simultaneous exposure to both addition and subtraction facts from grade 2 on may, in part, account for the non use of Addition by students. Children may not have had the opportunity to master relevant addition facts in order to use them for working out subtraction answers. Teachers' reports, that children were experiencing difficulties with addition as well as with subtraction number facts, reinforces this conjecture.

There is one additional consideration related to the selection of Choice by the grade 3 and 4 children. Many of their teachers in previous years had actively encouraged Counting Back on number lines or rulers to find unknown subtraction answers even when texts favoured the Addition strategy. It might be postulated, upon examining the Table 2 grade 1 and grade 2 data for Counting On (involving most of these same teachers), that not all children actually employed Counting Back as suggested. This assumption concurs with a growing body of research which suggests -

(1) that many young children develop the Counting On skill independent of instruction (e.g. Carpenter & Moser, 1983; Ginsburg, 1977); and
(2) that Counting On is easier than Counting Back for young children (e.g. Carpenter, 1983; Starkey & Gelman, 1982; Thornton, 1984).

Nonetheless, many of the Grade 3 and 4 students in the study were exposed to Counting Back, even if they actually used Counting On during their first years of school. It is not unexpected, therefore, to see them choose between these strategies, i.e. use Choice, in grades 3 and 4. There is a sense in which developing a more efficient counting approach may be more natural for students who are still unable to use Addition

reliable to determine unknown subtraction answers.

Discussion. Despite the evident popularity for Choice among many older students in the above study, it is a less efficient strategy than Addition and provides opportunity for error at each of its branching points. For any given problem a child must decide:

- Do I Count On or do I Count Back?
- What number do I start with when I count?
- What's my answer: the last number I say (as is the case when Counting Back) or the tally of the numbers counted (as is the case when Counting On)?

To further complicate matters many young children, especially those with learning handicaps, find it difficult to count back (Carpenter, 1982; Thornton, 1984). It is unclear whether the difficulty is due to specific learning deficits, or to the fact that the skill is often taught at the same time children are expected to apply it to subtraction (rather than as a prerequisite). Whatever the reason, it should be noted at this point that 60% of the third and fourth graders receiving Chapter I or Special Education support did not use Choice as a dominant strategy for finding answers to unknown subtraction facts.

This last result corresponds to results obtained by Thornton (1984) in a federally funded number fact project. Only 17% of the children receiving special education support for mathematics used Choice as a dominant strategy compared to 39% regular class grade 1 and 52% regular class grade 2 students in the same project. The special education students were drawn from two primary populations:
(1) learning disabled students in grades 1-4 and

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*Grant No. G008301694, Project No. 029KH30133, U.S. Dept. of Education, Office of Special Education and Rehabilitation Services.

(2) upper primary and lower intermediate mentally handicapped students.

Many of the special education teachers involved indicated they felt it best to only emphasize Counting On with their students. The relative success of the Choice strategy among younger students in regular classes may be attributed to the small step size of the instructional approach:

(1) Teach prerequisite counting skills (Counting On and Counting Back).
(2) Develop the subtraction concept.
(3) Teach Counting On to obtain answers to unknown subtraction facts. (Or, alternately, teach Counting Back. Teacher preference was honoured).
(4) Finally focus on Choice.

Discrimination activities like "find and do just the good Count Ons" were stressed during this latter stage.

Several questions arise from this discussion. Is Choice a natural stepping stone between low level counting and Addition Strategies? Or, given the inefficiency of the strategy and the apparent difficulty in its use by some students, is it possible to structure learning experiences which lead children to Addition earlier than now appears to be the case.

SECOND U.S. STUDY: MARCH-APRIL, 1984

To explore these questions Thornton (1984) carried out a pilot study that investigated the comparative effects of Addition and Choice strategies for finding answers to unknown subtraction facts. The design of the study was based on an earlier investigation carried out by Rathmell (1980). It also incorporated information (1) from the field testing of a Queensland, Australia subtraction program for children with special needs (Thornton & Toohey, 1983); and (2) from teachers in the federally number fact program in the U.S. referred to above (Thornton, 1984). Teachers in both the Australian and U.S. projects delayed work on subtraction number facts until related addition facts were mastered.

Sixteen third and fourth grade students were selected and randomly placed into one of two treatment groups; Choice (n=8) and Addition (n=8). Criteria for selection was the same as in the first study with the additional constraint that, upon entering the study, one of two dominant strategies was being used to determine unknown subtraction fact answers:

(1) Counting On; or

(2) Counting Back.

For each student, four unknown and two known subtraction facts were identified. Mastery of counting skills (both for counting on and for counting back) was established prior to carrying out subtraction instruction with the eight Choice group students. Likewise, mastery of addition facts related to the four unknown and two known subtraction facts was checked prior to subtraction instruction with the eight students in the Addition group. For both groups, reteaching on these prerequisites was carried out as needed. When the prerequisite learnings were mastered and shown to be intact after an 8-day retention period, students were individually guided through a series of five short tasks.

For the Choice group the goals were to help students discriminate when Counting On rather than Counting Back was appropriate for given facts; to provide practice in making an appropriate decision; and to provide corrective feedback when the less efficient path was chosen. For the Addition group the goals were to help students recognize an addition fact related to a given subtraction fact; "see" how it supplied the subtraction answer; and then identify, match or write addition facts which gave (or checked) answers to the two known and four unknown subtraction facts. At the end of the instructional session information on mastery of the four unknown facts was obtained. Additional retention information on these four facts was obtained 8-10 school days later.

Essentially no differences were found between groups on the level of mastery obtained. The strength of the study lie in its care with prerequisite learnings, which led to the easier understanding and use by children of the strategy being promoted during the

instructional session. In spite of limitations to the study (e.g. small n, shortness of instructional time; small number of unknown facts for each child), researcher observation of children's use of newly learned strategies provided support for the larger, somewhat modified investigation reported below.

THE AUSTRALIAN STUDY: MAY-OCTOBER, 1984

When an unknown subtraction fact is encountered, it is appropriate that children should be able to employ the most efficient strategy of which they are capable to work out the answer to that fact. While children tend to adopt dominant strategies for deriving fact answers, it is useful to learn, if possible, to shift to other strategies for certain facts -- e.g., to use patterns for 9s or pictures for doubles (Thornton & Toohey, 1983, 1984). Selecting relevant strategies for answering unknown facts is one aspect of metacognition (Masson, 1982), an active, self-monitoring approach that is being studied by psychologists to facilitate learning. Allardice and Ginsburg have claimed that such active learning can help children with learning difficulties to master basic facts.

The ability to use a strategy is the natural basis for it being selected by a child for obtaining an unknown subtraction fact answer. Previous discussion has documented the fact that most beginners use low level counting strategies to obtain answers to unknown facts. As children mature, one should expect that strategies for deriving fact answers should also mature. This indeed is another aspect of metacognition, the ability to reflect upon procedures being used and to modify them in light of new learnings.

Is Addition a viable strategy for children who have learning difficulties in mathematics? Or, if care is given to teaching prerequisite counting skills and to appropriate discrimination activities, is Choice a more suitable option? Reports from teachers in the federal number fact project (Thornton, 1984) and the results

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of the two studies above converge to suggest that if learning is carefully sequenced in small steps, and if work on subtraction fact memorization is delayed until related addition facts are mastered, then children may be able to move from low level counting into Addition -- by-passing the less efficient Choice strategy favoured by the older children of the first study above. The Australian Study was designed to further test the generalizability of that conjecture with children experiencing serious learning difficulties in mathematics.

Purpose

The major purpose of the Australian study was to determine, for children with learning difficulties in mathematics, the effectiveness of Choice and Addition strategies for finding answers to unknown subtraction facts. The timeline of the study (fig. 1) included five major phases:

(1) a pre-assessment interview;

(2) a pre-instruction phase during which pre-requisite learnings were checked and, if necessary, taught to mastery;

(3) an instructional phase during which either the Choice or Addition strategies were taught;

(4) a post-instruction check which examined the effectiveness of the previous lessons and of the additional instructional hints related to the Choice or Addition Strategy on the number of trials necessary to memorize unknown facts (those identified in (1), above); and

(5) a retention check which paralleled the task outlined in (4).

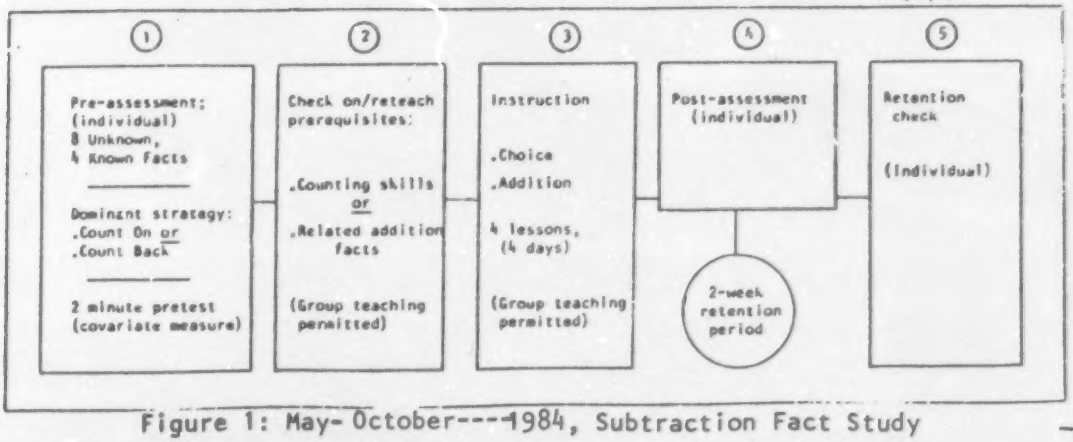

Figure 1: May- October---1984, Subtraction Fact Study

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The following questions are examined in the study:

(1) Are there any differences between Choice and Addition groups in the mean number of trials taken at the end of the instructional period to master unknown facts?

(2) Are there any differences between groups in the mean number of trials taken after the retention period to master unknown.

(3) Are there any differences which can be attributed to age, nature of handicap or previous instruction?

(4) Are children able to use the newly learned strategy during the post instructional assessment? during the retention check?

METHOD

Teachers Involved

Special Educators throughout Queensland -- Remedial Teachers and teachers in Special Schools/Units -- were invited to participate in the study. Interested teachers were then randomly assigned to one of two instructional groups: Choice or Addition. Ten mathematics seminars were scheduled during the twenty-ninth May and tenth August period at population centres throughout Queensland by the Queensland Division of Special Education. Each teacher received three days of intensive training on number facts which included two training sessions specific to the study. The research protocols, described below, served as the basis for this training. During one of the sessions teachers assumed pupil roles as one of the researchers demonstrated the teaching of the four instructional lessons for the treatment to which they had been assigned.

Children Involved

Children for the study were those taught by the teachers described above. They were drawn from two populations:

(1) Children in years (grades) 3 through 6 with learning difficulties in mathematics.

(2) Mildly intellectually handicapped (MIH) students enrolled in Special Education Units or Special Schools.

These students were assigned to part time remedial assistance in mathematics or to full time special education intervention on the basis of previous performance and testing. All could be

characterised as falling significantly below normal growth and achievement in mathematics, and were experiencing serious and consistent difficulties in learning mathematics.

Protocol Materials: Choice and Addition

To facilitate teacher training for carrying out the study and to insure uniformity within and between treatment groups on the length and nature of instruction, two parallel sets of protocol materials, (Choice and Addition) were prepared by the researchers. These materials served the following purposes:

(1) to detail the timeline for the study;
(2) to provide specific criteria for checking mastery of prerequisites;
(3) to provide details of the four instructional lessons;
(4) to provide details for post instructional assessment;

and,

(5) to provide details on the retention check.

Pre-assessment interviews conducted with children by the special education teachers were designed to identify the dominant strategy used by individual children for finding answers to unknown subtraction facts. Children who, on a regular basis -- i.e., for at least two out of three unknown facts -- used Addition, Choice, or other heuristic strategies for determining answers were excluded from the study. Pupils who used a three-step process, i.e., who relied on objects, tallies, or fingers to -

(1) show the whole;
(2) take away part; and finally
(3) count what's left,

were also excluded. Only children who -

(1) consistently used Counting On, or
(2) consistently used Counting Back,

were retained. Further assessment determined 8 unknown and 4 known subtraction facts for each child; and excluded from the study any child who correctly answered 35 or more facts on a two-minute, 54-item subtraction fact test.

A critical part of the design was checking and, if necessary, teaching prerequisites before beginning the four instructional lessons. Criteria for establishing mastery of relevant counting skills (Choice group) or mastery of related addition facts (Addition group) were provided in the protocol materials.

The instructional, intervention for each group consisted of ten tasks spread over the four days. The content for much of this phase of the study was drawn from Mathfact 2 (Thornton and Toohey, 1983), an instructional kit on subtraction number facts which was distributed to Remedial Centers and Special Schools during the ten training seminars.

Goals for the Choice group teachers for children who originally were Counting On are given below. Parallel goals for children Counting Back apply.

(1) Refine the child's dominant strategy (e.g. if the child was presently Counting On for all facts, the child was led to find and do "good" Count Ons). Develop the idea when using counters and reinforce often with the verbal prompt: "When the part you take is large, the numbers in the problem are close, so Count On".)

(2) Teach children to use Counting Back to find unknown answers. Example for 9 - 2: "9 -- 8,7. The answer is 7." Develop the idea with counters.

(3) Help children find and do "good" Count Backs.

(4) Help students discriminate, for any given fact, whether Counting On or Counting Back is faster.

(5) Provide practice in making the appropriate Choice and correctly using the strategy chosen.

Goals for the Addition group teachers were:

(1) Starting with known addition facts, to help children say or write related subtraction facts (and to illustrate this with counters).

(2) Given a subtraction fact and the related addition fact (at first, with answers given), to help children mark the digit in the addition fact that answers the subtraction problem (and to illustrate this with counters). Reinforce with the verbal prompt: "Subtraction is easy. When you know additon, you know subtraction, too".

(3) Given a subtraction fact and several addition facts in view, to help children answer the subtraction fact and match it to the related addition fact as a check. (Hide-and-seek cards (Thornton & Toohey, 1983, 1984) or counters were used to illustrate or check subtraction-addition relationships as needed).

(4) Given a subtraction fact, to help children say or write the answer and tell/write the addition that which checks/gives that answer. At this phase, addition facts were hidden from view until the subtraction answer was given. As in 3, hide-and-seek cards or counters were used as needed to establish/reinforce correct thinking.

(5) Provide practice naming the addition fact that checks/answers subtraction facts.

While other facts were occasionally brought into a lesson, teachers in both groups were directed to focus on the 8 unknown and 4 known facts for the children involved.

Upon returning to their schools, after a training seminar, teachers followed the guidelines/directions of the protocol materials for the treatment group to which they had been assigned. Actual work with children was therefore broken into ten overlapping segments during the June to October period. The starting date in each region was set for the Monday following the completion of the training seminar for that region.

Figure 1 summarizes the activity carried out by all teachers. Some moved through steps 1 and 2 more quickly than others, since this aspect of the study catered to individual differences in background

learning. Steps 3 to 5 were fixed: four consecutive teaching days, a short post-assessment check (described below), a 2-week retention period during which no instruction was carried out, and a final retention check. Assessment sessions involved individual work with students (except for the written pretest, which could be given to a group). Teaching sessions generally were group lessons. Group size ranged from one to five pupils; the typical group had two or three students.

Data gathered during the post instruction assessment provided information on the effectiveness of the Choice and Addition strategies for helping children find unknown subtraction answers. For each child the 8 unknown and 4 known subtraction facts were shuffled. Then, one by one, each card was shown and read to the child. A fact was considered to be "known" if the child could give the correct answer within two seconds. If children answered incorrectly, obviously used an inappropriate strategy, or hesitated beyond two seconds for a fact, teachers were instructed to provide an appropriate verbal prompt from the instructional lessons. If necessary, teachers were directed to count with the child (Choice group), show the related additon fact (Addition group), or provide other instructional hints the from the lessons to help children use the appropriate strategy to find the answer.

Each time all 12 cards were presented constituted as "trial". At the end of each trial, cards were reshuffled. Students were asked to continue to respond to the fact cards until they reached the mastery level of correctly answering each fact within two seconds on two successive trials. The number of facts correctly answered on each trial and the number of trials (to 12) taken to reach the specified mastery level were recorded on forms provided by the researchers. Information on whether or not the child appeared to use the strategy taught during the four lessons for figuring out unknown facts was also recorded.

The above procedures were repeated with each child as a final check after a two-week retention period.

DATA ANALYSIS

Ideally it would have been preferable to randomly assign children in the study to the two instructional treatments: Choice and Addition. In practice an approximation to this approach was achieved by randomly assigning teachers to the two treatments. This resulted in the students in each teacher group being assigned to either the Choice or the Addition treatment. Separate analyses will be performed for Remedial Resource and Special School students when complete details are available in October.

To test the differences between instructional strategies (Choice, Addition), age (Grade), and their interactive effects on the number of trials taken at the end of the instructional period to master unknown facts, a two factor fixed effects analysis of covariance will be performed using pretest scores as the covariate. The same form of analysis will be performed to test differences on the number of trials taken after the retention period to master unknown facts. Data from a questionnaire completed by teachers will supply qualitative data in addition to the quantitative data drawn used in the statistical analysis.

DISCUSSION

It is not possible at this time to make firm statements regarding trends in the data, since much is still unavailable. Information supplied by teachers to date, however, appears to be stable on one critical point. For older children with learning problems, Addition appears to be easier than Choice for deriving answers to unknown subtraction facts. One major reason for this appears to be the complexities of the decision making process in Choice. Delaying instruction until related addition facts were known may also have been a contributing factor.

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INTERVIEWS WITH MATH ANXIOUS ADULTS

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I. INTRODUCTION

Math anxiety has been defined and explored by many educators and researchers in a wide range of journals. The anxiety can create physical and/or psychological difficulties. The math-minded person, including teachers at all levels, may find it difficult to imagine how these difficulties can exist in relation to a subject with which they feel comfortable.

Who has math anxiety to such a degree that work or school or everyday life suffers? Many more people than you would imagine, unless you have carefully studied the topic. When does math anxiety start or when can it start? Math anxiety can start at almost any age after exposure to mathematics. The detection of math anxiety is usually delayed until some choice involving the use or non-use of mathematics is present. Do not confuse the onset of math anxiety with the detection of math anxiety.

What damage is done by math anxiety? The extent of the damage is almost beyond imagination. The participation by men and women in mathematics and related fields can be affected by math anxiety and by attitudes (Armstrong, 1982; Bleyer, 1980). The number of

adults who are handicapped by a fear of mathmatics is staggering (Hilton, 1980). A problem of this magnitude cries out for study.

Both men and women have math anxiety. Women's avoidance and fear of mathematics has been studied perhaps to a greater degree, but whether one group or the other is more prone to math anxiety is a question that has yet to be answered conclusively.

What are some of the factors that contribute to math anxiety? The pressure of a timed test can constrict the free flowing thought processes needed to solve problems (Blum & Givant, 1980; Morris, 1981). Test anxiety is a topic in its own right and can appear within any subject matter or testing situation. Confidence in one's ability to learn mathematics has been mentioned by many researchers (Dolan, 1983; Fennema, 1981; Wolleat, Pedro, Becker, Fennema, 1980) as well as attitude toward mathematics (Eldersveld, 1983). Fear of intimidation within a mathematics classroom can create or increase math anxiety (Brunson, 1983). The mathematics curriculum itself is coming under fire for introducing topics in the elementary school before the students are developmentally ready for them, for example the topic of fractions (Hulse & Meyer, 1982). Interaction between instructional method and math anxiety has been studied but results are not conclusive (Clute, 1984). Reading ability has been linked to math anxiety in several ways (Tobias, 1980). Reading mathematics is totally different than reading in a reading class, in everyday

life situations, and in most other subject areas. A good reader in history or political science may not be a good reader in mathematics. Mathematics must be translated into abstract symbolism and a literal translation may not be the best way to do it, just as in translating from one language to another.

How is math anxiety detected? Many researchers use rating scales and tests of various types. The results from The National Assessment of Educational Progress have been analyzed for many factors. If test anxiety is related to math anxiety, could the results of a test or rating scale affect the true picture of math anxiety? It is also possible that the persons being tested or rated may answer the way they think the researcher wants, rather than the true way they feel (Carpenter, Corbitt, et al, 1980). The use of other devices such as questionnaires, individual interviews, written personal histories, and group discussions may present a better picture of the status of math anxiety.

II. BACKGROUND

Triton College is a community college located in the suburbs west of the city of Chicago, Illinois, USA. As a community college it is mandated by state law to offer the first two years of university courses, technical programs of one or two years duration, adult education courses, and developmental education for adults

lacking elementary or high school skills. The students who attend a community college have widely differing backgrounds and abilities. Many students who attend Triton College are working full-time and take a limited number of courses each semester. The enrollment figure of 26000 includes approximately 5000 full time students.

The Mathematics/Computer Science Department offers a range of courses from arithmetic to differential equations. The interviews were conducted with students enrolled in a course called College Algebra, MAT 110. The material covered in this course is also offered in the traditional high school curriculum in the third or fourth year. Many students have not taken the course in high school and many more have taken it and performed poorly. Entering college and looking at the requirements for almost all majors is a shock for these students. College Algebra becomes a necessity before their course in statistics, finite math, linear algebra, or calculus.

During the school year 1983-84, I taught College Algebra in a format consisting of a large lecture (120 students) 3 times a week and small discussion groups (20 students) twice a week. My full teaching load was taken up by this one course because I gave the lectures and led the discussion groups.

III. PROCEDURE

The first week of the semester was devoted to learning how to

read and solve logic puzzles and problems. Since reading mathematics is a different skill than reading in general, students who can be shown the difference can learn to overcome the tendency to read mathematics by skimming. A questionnaire distributed during the first class contains the usual questions about background, major field of interest, and a few rating type questions such as 'I love-hate mathematics' and 'I am afraid to take tests--yes--no'. While the questionnaires were being completed by the student, I gave a brief description of test anxiety, math anxiety, and the need to alleviate these anxieties for a better learning environment. In the discussion groups, these anxieties were explained in more detail so that students could decide if they themselves were so afflicted. I announced my availability to discuss these anxieties and/or any other difficulties with the mathematics class on an individual basis.

A few students came immediately for help or just to talk. Later more students came, sometimes alone, sometimes in groups. During the semester, as a writing assignment for the class, the students wrote a short paper on math anxiety, if they had it or not, when it started, and how serious it was. Finally, individual interviews were set up as an adjunct to the papers.

The results of this study have not been studied statistically because the data is not quantitative in nature. The study looks

at math anxiety through the eyes of people, a humanistic view. Students who have shown the greatest anxiety are quoted directly because they are the victims and they can perhaps shed the greatest light on this problem.

IV. RESULTS

From the papers, individual interviews, and group discussions, the following items were frequently mentioned as being the cause of students' math anxiety.

(1) Shyness--Being shy prevents a student from asking questions which in turn can hinder understanding of a concept.

(2) Test Anxiety--A student can know the material but becomes anxious during a test and can not answer the questions. If this occurs only during mathematics tests, math anxiety increases.

(3) Textbook--If a student can not read the textbook, learning is hindered.

(4) Goal Setting--Unrealistic goal setting creates the situation in which a student is doomed to fail.

(5) Flash Cards--The pressure in elementary school to succeed in situations where flash cards are used can be overwhelming.

(6) Chalkboard--Working problems at a chalkboard allows all others in the class to see errors or hesitations in solutions.

In the interviews with the math anxious adults, shyness came up as a cause of the anxiety, in almost every conversation. Those who did not mention shyness felt math anxiety only during tests, possibly not a true case of math anxiety. As a surprise, the use of flash cards in elementary school was also mentioned by almost all the adults. They felt the pressure put on them as children to perform in a timed situation and/or before their peers was the start of math anxiety. Some adults also mentioned the pressure of having to go to a chalkboard and perform before their peers as contributing to their math anxiety. The difficulty of reading a mathematics textbook at all levels was frequently mentioned. Few teachers take the time to teach reading in a mathematics context, possibly because the teachers are unaware of the problem.

Unrealistic goal setting can increase math anxiety in an adult, without the awareness of the adult. Adults who enter a college expect that they can perform well. They may realize that all subjects may not be as easy, but certainly they can succeed. They do not realize that outside obligations can drain them of the time and energy necessary to succeed to the level of their expectations. If adults can learn to set realistic goals based on their abilities, available time, and energy, math anxiety can not arise because of this factor. The adult may still have the anxiety in relation to some other cause.

Events occuring during the elementary school years can initiate math anxiety. This anxiety, if unchecked, can increase in intensity until in adulthood, the study of mathematics becomes impossible. Children who skip a grade, change schools frequently, or have traumatic events within their home lives can arrive at adulthood with severe cases of math anxiety. It doesn't always happen, and perhaps further research can enlighten us.

The puzzle of math anxiety has many pieces. Each adult has a different puzzle, but some pieces are in common. Helping an adult to see the pieces and put them together can be a first step. Many colleges and universities now offer seminars and workshops for women with math anxiety. Men should not be overlooked. They can and do have math anxiety, but frequently are reluctant to admit the problem exists. Learning how to read and study for a mathematics class are skills that can be taught to adults. Techniques to use in test taking as well as methods of relaxation can ease the anxiety of many adults.

If math anxiety is such a serious problem today, two things must be done. We must help the persons so affected, whether adults or children, so that their lives are not devoid of the study or use of mathematics. We must also find and correct the teaching procedures that can cause math anxiety, in one child or in all children.

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TEACHER TRAINING FOR DIAGNOSIS AND PRESCRIPTION IN MATHEMATICS EDUCATION

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INTRODUCTION

It is the intention of this paper to describe one attempt to narrow the gap between the theoretical knowledge about mathematics education, as expressed by cognitive theorists, social psychologists and educational researchers, and teaching practice in the classroom. In many ways the title of this paper is inappropriate although it closely resembles the original name of the teacher training course under discussion. In fact, the course uses a model of problem solving as its structural framework rather than a medical model. Nevertheless, the attitudes and beliefs implied by terms such as 'diagnosis' and 'prescription' are central to the following discussion.

The concern of educators and educational policy makers about the extent to which mathematics education, in its present form ,is appropriate as a means of meeting the needs of future citizens in an increasingly technological society, has been well documented. In England and Wales, the report produced by the Committee of Enquiry into the Teaching of Mathematics in Schools under the chairmanship of Dr. W. Cockcroft (1982) suggests that considerable changes in the processes of mathematics education are needed. After making detailed recommendations for changes in mathematics curricula in order to better meet the needs of pupils, the report suggests that: "In order to present mathematics in ways that we have described it will be necessary for many teachers to make very great changes in the ways in which they work at present. This may not be easy....." (paragraph no. 465).

It appears, in Australia at least, that many teachers are painfully aware of the inadequacies of many mathematics curricula and are uncertain of their ability as professionals to meet the needs of the pupils in their charge. The House of Representatives Select Committee for Special learningdifficulties Report (1976) notes the negative attitudes of most teachers as an educational problem in mathematics.

The Auchmuty Report (1980) and the report of the Senate Standing Committee of Education and the Arts (1981) make particular reference to the lack of mathematical competence and the negative attitudes of primary school teachers. A recent survey carried out by the author of this paper suggests that secondary mathematics teachers feel unable to meet the learning needs of as many as 20% of students entering High Schools.

Many classroom teachers express feelings of powerlessness in relation to mathematics teaching. These feelings appear to be associated with at least two major aspects of their own experience. Firstly, many of these teachers are influenced by views about mathematics and mathematics teaching that might be termed knowledge only in the socialogical sense. That is, these views represent a set of beliefs that are widely held by a large group of people both within and outside the teaching profession. Such beliefs result in teaching behaviour, learning behaviour and parent attitudes that tend to maintain the status quo and make educational change difficult. Some of these beliefs are listed below.

- Mathematical ability is strongly associated with intelligence. Perhaps because of this, there is a wide acceptance of failure in mathematics since this is usually attributed to the characteristics of the learner rather than to the quality of educational experience.
- Mathematics is seen as an antonomous realm of knowledge that is divorced from empirical experience. Logic, rationality and truths derived from deductive reasoning are considered to be self evident and to require no further explanation (e.g. Kuhn (1962, 1977))
- Socialogical variables such as race, class or ethnicity, when considered as causes of low achievement. Remedial action usually involves attempts to change the student to fit the education system rather than a reappraisal of the match between the characteristics

of the child and the characteristics of the curriculum.

Too often, these beliefs lead to educational practices in the teaching of mathematics that would no longer be acceptable in other disciplines. For example, in mathematical problem solving it is the teacher who sets the conditions of the problem and judges the corrections of student thinking. In no other curriculum area is there usually perceived to be one right way to reach one correct solution. Yet, this is the commonly held view of school mathematics. Students perceive the task of learning mathematics as "renumbering how to do it" and are placed in an anxiety provoking and extremely dependent learning situation.

Most teachers have themselves learnt mathematics at school as a series of facts and procedures to be numbered. As a result of their own limited experience in elementary mathematical thinking, they find it very difficult to apply teacher training in the theoretical aspects of the teaching/learning process to the teaching of mathematics. Too often, when confronted with a student experiencing difficulty with the usual school curriculum they feel powerless to make a professional judgement about why such difficulties are occurring. And, because mathematical knowledge is revered as a 'truth' they are anxious about making changes in curriculum content of teaching procedures.

Training for a Wider Range of Teaching Behaviour in the Mathematics Curriculum

The presentation of any knowledge to school children in ways that are likely to make it a functional tool for solving problems of the future, is not merely a matter of making students aware of such knowledge. Experiences must also be provided in the use of knowledge if students are to develop higher order thinking skills, confidence in making critical judgements and flexibility in reasoned problem solving in their own lives.

Because of the educational attitudes, described above, and past learning experiences, many teacher trainees (and indeed most teachers) also need experience in the use of mathematical thinking to solve salient problems before changes in teaching practice can reasonably be expected .

It seems important that any course for teachers should provide more than information about cognitive development as a rationale for teaching behaviour in mathematics. Recent research by the author (1)* suggests that although most teachers in early grades are aware that they should use concrete materials and activity based programmes very few understand why and most consider such activities as 'props' to be used only with childreb who are not yet ready for symbolic arithmetic computation. A shift is needed in training on problems to include an understanding of the principles of social psychology as these relate to mathematics learning and some experience of their application. For example, as well as providing information about the work of such researchers as Skon et al (1981) and others it is necessary for teachers to experience for themselves a wider range of learning conditions in mathematics as models for their own classroom behaviour. Also, because school assessment in mathematics is usually rather narrowly based, teachers need experience, in the development of procedures that are more appropriate to the needs of those large numbers of students currently experiencing difficulty with mathematics in school. In particular, since many teacher trainees have learned mathematics by memorizing algorithmic procedures, they are particularly unskilled at the more holistic visual representations that are useful for communicating many relationships and patterns in mathematics and more appropriate for some cognitive styles.

Such a course might aim to increase teacher abilities to recognise and manage those aspects of a childs learning environment that are defective or inappropriate. Consideration of the whole learning process rather than the individual characteristics of the students would be desirable. Teachers may then be in a position to use such relevant variables as cultural expectation, self concept and role to enhance the learning process.

With the above points in mind the post graduate course "Diagnosis and Prescription in Mathematics" was designed with emphasis on problem solving and professional decision making. The course formed one quarter of the requirements of the Graduate Diploma in Special Education offered at the Canberra C.A.E. This diploma is available to experienced teachers at the early childhood, primary and secondary levels. The initial course involved two three-hour workshops and approximately two hours classroom practice each week for one semester.

During classroom sessions the teachers gained experience in assessment, construction of appropriate individualised curricula and the process of matching teaching procedures to the needs of the child (see Fig 1 below).

The workshop experiences and classroom assignments were designed to cater for the following major needs of the group:

* Positive feelings and increased confidence about mathematics and their ability to teach mathematics.
* Confidence in their ability to make judgements about individual students needs and adapt teaching practices to meet these needs.
* Experience in all aspects of problem solving.
* Experience in teaching and learning mathematics in more egalitarian conditions,
* Understanding that knowledge and competence in other curriculum areas has application in mathematics education.

The workshop sessions were structured to provide three major kinds of learning experience.

ASSESSMENT AND TEACHING IN MATHEMATICS

Begins with the basic premise taken from N.C.T.M. and A.A.M.T. statements on basic skills that the chief purpose in studying mathematics is the development of problem solving skills.

COURSE STRUCTURE

Steps in Problem Solving	Perception of Problem	Understanding Problem Conditions & Relationships	Motivation to Proceed	Select General Strategy	Select and Implement Tactics	Evaluation Solution	Explain or Modify Consequences
Characteristics of Learner Learning Experiences About : Characteristics of task Characteristics of Learning Environment	Initial input on reasons for learning difficulties. . Information processing. . Developmental difficulties. . Evaluation procedures. . Attitude to maths. . Cultural background. . Teacher behaviour. . Learner behaviour. . Curriculum Design. . Expectations.	Extensive library research. Information exchange based on teaching experience. Exploration of relationships between learner/teacher/task. Mostly small group discussions - intergroup argument. Reports on Assessment.	Increasing demand for independent judgements and ability to express and justify a professional opinion. Critical evaluation of available forms of assessment.	Group discussion of possible strategies. Practical workshops on : Environment. Task Adaptation to meet specific learner needs.	Reporting Workshops. Small group discussions according to interest and expertise. Development of Assessment Reports. Program Advice. Strategies for Teachers.	Increased group participation in decision making about means of communicating with teachers, schools, parents.	Critical evaluation of learning outcomes - group discussion of evaluation procedures - decision making about modification of programs.
Assess Characteristics of Learner Teaching Experiences Adapt characteristics of task Consult and where possible modify characteristics of learning environment.	Assessment of learner abilities, and attitudes, concept of mathematics, problem solving strategies. Observation and evaluation of classroom behaviour of both teacher & student. Critical evaluation of cognitive behavioural demands of mainstream curriculum content. Assessment of home environment where relevant.	Make professional judgements about the implications of data collected about : a) ecology of classroom, b) needs of student, c) attidudes, expectations of parents. d) test results.	Accept responsibility for making decisions based on data collected. Assume limited "expertise" about needs of students.	Devise Individual Programs. - content, - teaching procedures, - task adaptation to learner characteristics. Maximise match between Learner characteristics - Task demand - Environment characteristics. Parent counselling & involvement in intervention strategies.	Decide on Priorities - short and long term goals. - implement a series of intervention sessions.	Formative Evaluation of Outcomes Suggest modifications to : - intervention programme, - classroom procedures and behaviour management, - parent role.	Construct assessment and learning reports for schools. Consult with classroom teacher about ongoing program. Construct appropriate form for communication with parents where applicable.

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Firstly, teachers were encouraged to exchange information about learning difficulties, teaching procedures and assessment. Although a theoretical framework was initially provided by the author, information exchange and discussion was increasingly initiated and implemented by the teachers themselves. A library research tutorial was an important means of increasing independence in seeking information.

Procedures for assessment and teaching were applied by each teacher to selected pupils who were experiencing difficulties with their mathematics learning environment. Each teacher assessed and then developed an individual program for at least two pupils. The results of these activities became an increasing component of the information exchange between teachers. In particular, learning difficulties came to be seen as resulting from the characteristics of the child. Teachers began to understand that classroom procedures could be changed to suit the child with profitable results. For example, visually impaired children and children with a strong preference for auditory information processing often benefited considerably from the manipulation of large simple objects to illustrate mathematical ideas. In other cases childrens attitudes changed when they made up the questions and the teacher found the answers.

Workshop sessions were structured so that the experiences of the teachers mirrored the steps in problem solving (see Fig 1). Thus the assessment and teaching of pupils was posed as a professional problem - their problem. Initial activities were aimed at broadening teacher perception of the possible variables involved in learning difficulties, how these related to each other and to the learning environment. Later, when the problem was more clearly understood and related to existing teacher knowledge of teaching procedures in other curriculum areas, possible strategies were discussed for altering the negative aspects of a child's learning environment. In these discussions the emphasis was placed on principles of procedure for developing understanding of mathematical ideas and concepts rather than encouraging the role learning of facts and operational routines.

Methods of adapting communication strategies and task demands to match the information processing abilities of children were also considered.

Teachers finally selected teaching strategies and content appropriate to the children in their care, implemented and evaluated individualised programs.

Workshop sessions provided experiences in modelling, role play, debate, small group discussions and (especially) individual and group decision making. Past learning experiences in mathematics had made many teachers most anxious about any demand for a judgement in this field. They rather expected the author to correct them. The discovery that they were able to make professional judgements about mathematics learning with as much competence as those made with confidence about language arts skills was an important experience for many. The experience also heightened their awareness of the affective aspects of their own experience in learning mathematics. They had discovered for themselves that a more egalitarian atmosphere in which their opinion was valued made it easier to think about complex problems.

As the semester progressed some teachers in the group developed a known "expertise" about the characteristics of certain learning disabilities or misconceptions about mathematics. All teachers increased in confidence and willingness to suggest and implement solutions to learning difficulties.

Personal Interactions and Emotional Responses

The personal interactions and emotional responses of individual teachers were similar to those described by Block (1982) in his description of the affective components of the Piagetian notions of assimilation and accommodation or the changing of old ideas and assumptions in the face of new information. Care was taken initially to link new ideas with the known past experience of the teachers in the group. As a result, there were initial expressions of relief as it became obvious that some of the expectations based on anxious past experiences with mathematics learning were not warranted in this course.

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There followed a period of anxiety, confusion and sometimes even resentment as they discovered that a more egaletarian workshop atmosphere as well as being less judgemental also demanded that they reevaluate old teaching habits and assumptions for themselves. These feelings were expressed in such ways as; 'You've made me question everything I've ever done!', 'Why can't you tell us what to do? - It's such hard work deciding for ourselves' 'How will I know which article to report to the group?'

Surprisingly, most of these feelings were a result of the teaching procedures, particularly the demand for considerable independence, rather than the mathematical content of the course. The role of a helping teacher became a major issue for discussion. As the confusion abated and each teacher began to accommodate the new ideas, experiences and information and to develop a personal rationale for teaching and confidence based on experience in experimenting with new procedures, they began to value independent learning for themselves and also for the students in their care. Some care was required to ensure that the feelings of discomfort did not become too great. So, sometimes a workshop became a lecture and needed information was supplied and teachers were only required to listen.

As teachers came to know more about the children in their care, and gained experience in weighing their judgements against the information and advice of other members of the group, enormous leaps in understanding and confidence occurred. The process was mirrored in their own teaching. For example, the following comment appeared in one case study:

> "Kate lacked confidence and kept asking for help. When this was delayed she managed to work out problems for herself and was pleased with this."

CONCLUSION

The above report is intentionally descriptive since the course described is still very much in the developmental stages. The application of a process curriculum design (in the sense used by Stenhouse (1976) has been most successful as a means of developing personal rationales for assessment and teaching in mathematics. The major positive outcomes for course participants were in the affective domain and increased professional independence due to better problem solving skills. As they began to express mathematical ideas with students rather than 'tell them how to get it right' they felt much more comfortable. Many of them also learnt a great deal in the process.

The increase in confidence and professional assurance of the numbers of the group were notable. In particular, once teachers had experienced the satisfaction and sense of personal control of independent problem solving for themselves, they understood the need to foster independence in their pupils. There was a purposeful shift towards more egalatarian teaching roles as a means of developing higher order intellectual activity and problem solving skills.

The initial one semester course caused some difficulties for some students who had insufficient time to develop and practice new thinking skills. This was particularly evident in the case of using visual thinking and visual media for communication. As a result of these difficulties, the course will in future be run as two units over two consecutive semesters.

There are indications that for many students the changes in teaching behaviour are lasting. At least two major mathematics curriculu, developments have been initiated in schools by students, as a result of the course experiences. Also, a professional support network has been formed as a forum for continued information exchanges about issues and procedures in mathematics education. The members of this group of teachers are currently producing a series of videotapes for professional inservice on remedial instruction.

The following statement was made by a teacher in the final assignment. It is an eloquent statement of the attitudes to teaching expressed by many course participants.

"Looking back on the experience now, I think I found it most rewarding - more a learning experience for me although I know they also benefited."

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PROJECT IN THE MATHEMATICAL EDUCATION OF RE-ENTRY ADULT LEARNERS

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BACKGROUND

A few years ago, the Inner London Educational Authority recognised a need to encourage those who had not succeeded in their formal schooling to re-enter the educational network. In particular, this initiative was designed to offer a new educational opportunity to members of minority groups and to adult women, who were under-represented in tertiary education. The courses so designed are called ACCESS courses as their purpose is to provide access to specific higher education. These courses are run at Colleges of Further Education which link with the Higher Education establishment which will receive those students who complete successfully.

All students on Access courses have in common a need for study and communication skills. More recently, computer literacy has also become a central feature of these courses. In addition, any Access course linked to a Bachelor of Education degree giving qualified teacher status must meet Department of Education and Science requirements on equivalence in standard to a General Certificate of Education Ordinary level pass in English and Mathematics. Many Further Education Colleges submit their Access students for the General Certificate of Education 'O' level examinations in these two subjects, but this is not required as long as equivalence in standard can be demonstrated.

Avery Hill College is linked with four F.E. Colleges for access onto the B.Ed. (primary) and B.Ed. (secondary - Youth and Community, Craft, Design and Technology). All of these students must demonstrate satisfactory attainment in Mathematics.

In January, 1983, on becoming head of Mathematics at Avery Hill College, I was involved in planning a new Access course for B.Ed. (primary) and B.Ed. (secondary Youth and Community) students at Hackney College of Further Education. I was aware that many access courses had a history of poor achievement in mathematics which confirmed students' previous failure in the subject. I was also aware of initiatives which had been taken in mathematics anxiety clinics and had a personal research background in problem solving. It seemed appropriate, therefore, to construct an entirely different approach to the mathematical education of these re-entry students.

The Course

Called Mathematics: Thinking and Action, the course was designed to:

(a) capitalise on the adult experience and motivation of those taking the Access course;

(b) present mathematics as an enquiry-based subject requiring active involvement;

(c) offer learning experiences which build confidence and a positive self-image;

(d) encourage students to change their perspective on the learning and teaching of mathematics.

Changing these students' perspective on the learning and teaching of mathematics was seen as a pre-requisite for prospective teachers.

The course has two components, independent study packs and group problem solving. The set of thirteen independent study packs consists of five on Number, entitled Numbers in Use, Whole Number Patterns, Bits and Pieces, Percentages, Ratio and Proportion; three on geometry; two on the use of graphs and statistics and three on algebra. The packs were created out of carefully selected existing curricula resources boosted by specially written materials. The choice of what to include reflects the aims and objectives of the course and suitability for adult learners. The format of each pack is the same.

The first sheet consists of a flowchart which guides the student in the use of the pack.

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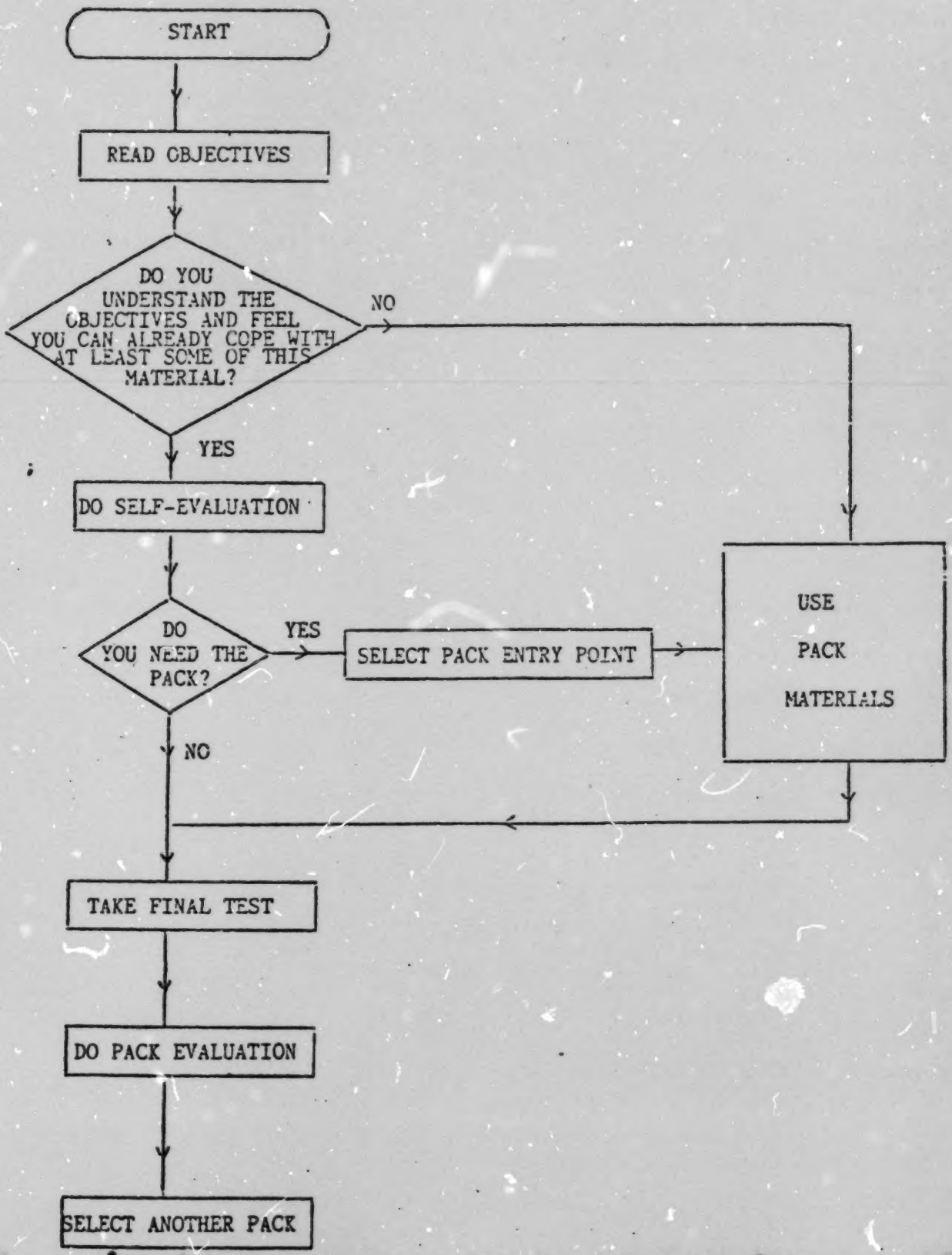
"FLOWCHART"


START
READ OBJECTIVES
DO YOU UNDERSTAND THE OBJECTIVES AND FEEL YOU CAN ALREADY COPE WITH AT LEAST SOME OF THIS MATERIAL?
NO
YES
DO SELF-EVALUATION
DO YOU NEED THE PACK?
YES
SELECT PACK ENTRY POINT
USE PACK MATERIALS
NO
TAKE FINAL TEST
DO PACK EVALUATION
SELECT ANOTHER PACK

If a student is unable to cope with any of the objectives, she is advised to work through the complete pack.

If the student does not feel the need to study all the materials in the pack as some of the objectives deal with familiar ideas/methods, she is directed towards completing a self-evaluation sheet.

The set of objectives for the pack constitutes the second sheet. In the Numbers in Use pack, for example, this sheet states:
This pack will help you to:

1. State the value of a digit in a decimal number.
2. Use place value to multiply and divide by powers of 10.
3. 'Round off' quantities to a reasonable degree of accuracy.
4. Approximate using the four operations (addition, subtraction, multiplication, division).
5. Calculate accurately using the four operations.

The list of objectives is followed by a list of materials which are the resources associated with each objective. These include computer programs, videotapes, as well as manipulables and activity sheets.

The next sheet, the self-evaluation, has a set of questions which correspond to each of the pack objectives, together with a final question requiring the student to demonstrate understanding by reacting to a frequent pupil response. The team refers to this as the 'Jane says' question. It is included in the self-evaluation to test understanding in a context appropriate to aspiring teachers.

As an example, there follows the Self-Evaluation from the Numbers in Use pack.

SELF EVALUATION 96

NUMBERS IN USE

PLEASE DO NOT WRITE ON THIS SHEET

(1)

(a) What does the six mean in each of these numbers ?

£6214 _____ 9.6 cm _____ 76 trombones _____

(b) Write this number in figures :

four hundred thousand and twenty-four _____

(c) At the entrance to a football pitch the turnstile automatically counts the number of spectators passing into the grounds.
The counter currently shows 19999. How many people will be inside the grounds after the next person has passed through the turnstile ?

_____

(2)

Use the number

60.28

to write down the answers you get when you :

(a) multiply by 100 _____

(b) divide by ten _____

(c) multiply by 1000 _____

(3)

There were 63481 people at an important football match.

Write this figure :

(a) to the nearest hundred _____

(b) to the nearest thousand _____

(4) Can you say at a glance which of the given numbers is nearest to the right answer ?

378×2.4

600 800 8000

Now do the same for this division : £9458 ÷ 52

£18 £108 £180 £200

(5) Give exact answers to the following:

(a) Add 3.2 , 37.04 , 53 and 0.275

(b) Subtract 13.46 from 21.3

(c) Multiply 37.64 by 5.8

(d) Divide 31.62 by 1.2

Jane was given this question in her maths class :

PENCILS ARE SOLD IN BOXES OF 100 AND COST £3.49 A BOX . HOW MUCH DOES ONE PENCIL COST TO THE NEAREST PENNY ?

Jane did the division .

$$100 \overline{)£3.^{3}4^{49}9^{90}0} \quad 0.0349$$

She explained ; "Now I've got an answer £0.0349 . That's the same as 3.49pence. That's the cost of one pencil, but I've got to change it to the nearest whole penny."

"Well, I'm starting with 3.49 so I'd better get rid of the 9 first.

The rule teacher said to use was if the number is 5 or more then you add one to the next number. So 3.49 becomes 3.5

Now I've got 3.5p. That means I add another one because it's still 5 or more so 3.5 changes to 4.

One pencil costs 4p to the nearest penny."

What would you say to Jane ?

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A student chooses to enter the pack at the section which corresponds to the objective which presented difficulties. Lack of success with question 2 on the self-evaluation, for example, indicates the use of section 2 of the pack. The pack materials are sequential so entry at section 2 infers completion of all remaining sections in that pack.

If the student does not feel the need to work that pack, or on completion of the pack, she is tested. The test, an example of which from the Numbers in Use pack follows, consists of a set of questions at least two of which have been taken from former G.C.E. 'O' level mathematics papers, thus ensuring that the appropriate standard is demonstrated. To pass the test, the student must gain a minimum of 65%. Resits are permitted although it was hoped that students would not choose to take the test before they felt ready to succeed. There is no choice of questions on the tests.

TEST

(1)

Calculate :

(a) 2.1 x 10

(b) 67 ÷ 100

(c) 0.002 x 100

(d) 87.9 ÷ 1000

(e) 0.37 x 10

(f) 2.3 x 100

(2)

Evaluate exactly

(a) 0.3 + 0.05

(b) 0.4 x 0.004

(c) 500 ÷ 0.05

(3)

Calculate the exact value of

(a) 27 x 0.016

(b) 0.027 x 180

(4)

In September 1979 the tourist exchange rate in Switzerland was 3.40 francs for £1 sterling . Calculate

(a) How many Swiss francs could be bought for £45

(b) The cost , in pounds sterling , of buying 74.8 Swiss francs.

(5)

Calculate

(a) The cost of fifty $8\frac{1}{2}$p stamps

(b) The number of $6\frac{1}{2}$p stamps that can be bought for £13

(6)

Choose the correct answer.

The value of 0.1 ÷ 0.01 is

(a) 100

(b) 10

(c) 1

(d) 0.1

(e) 0.01

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( )
Which of the following degrees of approximation are always the same ?

1 Answer to one significant figure

2 Answer correct to one decimal place

3 Answer to the nearest tenth

Choose ONE of the following

(a) 1 and 2 only

(b) 1 and three only

(c) 2 and 3 only

(d) 1, 2 and 3

(e) none of them

(8)
Write down

(a) 4.7248 correct to 3 significant figures

(b) 0.01177 correct to 3 decimal places

(9)
The value of $\frac{16.9 \times 0.635}{3320}$ estimated correct to one significant figure is

(a) 0.0003

(b) 0.003

(c) 0.03

(d) 0.3

SOURCES

qu.2 AEB paper 2 Jun 77
qu.3 AEB paper 2 Jun80
qu.4 AEB paper 2 Jun 81
qu.5 Cambridge syllabus C paper 1 summer 78
qu.6 London syllabus D paper 1 Jun 76
qu.7 London syllabus D paper 1 Jun 77
qu.8 AEB paper 2 Jun 81
qu.9 AEB paper 1 Jun 80

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As the G.C.E. 'O' level paper does not necessarily contain questions on every part of that syllabus and a pass can be obtained with approximately 40% of the marks from a choice of questions, it was felt that any student completing all these thirteen independent study packs and gaining at least 65% of the marks on each test would have amply demonstrated understanding and facility to 'O' level standard.

The intention of the independent study packs was to ensure student-driven learning in which the decisions controlling the content and the pace rested with the student. It was, however, hoped that enough students would be working on any particular pack at the same time to encourage collaboration, discussion and group enquiry. The tutor's role was to establish and support this style of working, on a one-to-one or one-to-many basis, as required by the students.

The group problem solving sessions, the other component of the course, were tutor-initiated and offered opportunitites to use games, puzzles and investigations and to reflect upon the processes that were successfully employed. A positive atmosphere encouraged students to recognise and consciously to utilise their problem solving processes. The conscious reflection was directed at building confidence in two ways:

by encouraging recognition of their mathematical thinking skills within their current repertoire of behaviour, and then

by promoting the use of these skills to encourage a problem solving style of learning when using the packs.

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The course was constructed by a team consisting of three tutors from Avery Hill College, Leone Burton, Geoff Coggins and Angela Walsh, the tutor responsible for implementing the work at Hackney College of Further Education, Jean Connor, the Mathematics Advisor from the London Borough of Croydon, Ruth Townsend who was interested in the potential of a course such as this in schools and Colleges, and the Head of Mathematics in a London secondary school, Mike Hall, who was closely involved with the development of materials for the I.L.E.A. Secondary Mathematics Independent Learning Experiment (SMILE) and who was also interested in the potential of this course for independent adolescent learners.

Evaluation

The course has been pilotted throughout the academic year 1983/84 in two ways, as the mathematics component of the Access course at Hackney College of Further Education (initially 11 students) and as a return-to-study course not associated with guaranteed access at Avery Hill College (initially 25 students). A selection of packs was also made by a post-graduate Certificate of Education student who used them with ten fifteen-year-old boys attempting the Certificate of Secondary Education in Mathematics and with nine fifteen-year-old girls who were attempting 'O' level Mathematics. All of these pupils were at Mike Hall's school. Nine Hackney students completed the course, eighteen Avery Hill students remained throughout the year and seven have chosen to return for a further year from the forthcoming September. At Hackney the students were mixed in age, sex and ethnic origin. One student had completed an 'O' level mathematics course previously but had failed to sit the examination. The remainder had little mathematical background. At Avery Hill College, all the students except two were female and the average age was 38. Their background in mathematics was very limited except for one student who had passed 'O' level but was very insecure and one who had worked at this level before but not attempted a formal assessment. At Hackney students were offered three hours per week on the packs and one and a half hours per

week group problem solving. In the third term, time was increased by two hours per week but problem solving time was eroded because of the pressure to complete the packs. Most students claimed to spend about five hours per week of their own time on the packs, in addition. At Avery Hill College, one session per week of two hours was scheduled for each of two groups. Those students who elected to attend both sessions obtained four hours per week of contact time. The packs were so popular with these students that they exerted strong pressure to abandon time devoted to group problem solving. They disliked the six day gap between the once weekly sessions and asked to borrow packs to take home.

The Use of the Materials

The first pack, Numbers in Use, was too vast, especially for a starter pack and completion time varied between sixteen and thirty-six hours, in contrast to subsequent packs which took approximately eight hours. Students' confidence levels were too low to use the self-evaluation as intended and they chose instead to work the pack from beginning to end. A few Hackney students opted to take the test before they were ready and did not succeed in gaining 65%. They had to re-sit. This was unexpected however discussion with the students concerned indicated that the ability to judge readiness is, in part, a function of experience with learning. Consequently, the tutor adopted a responsibility to guide and suggest further work where appropriate with the early packs. As expected, students' ability to assess their own readiness and adopt responsibility rapidly developed.

Students adopted very different rates of progress with the packs which meant that in small groups there were few opportunities for group work and discussion. Students did refer to one another where guidance was required, a slower student turning to one who had already completed that section of the pack causing difficulty but this was a helper/helped relationship rather than a group learning situation. Where opportunities for students to extend and reinforce

their learning by playing games or attempting small investigations, these were too often ignored or avoided especially in the early packs, because students would have had to break away from different sections of the pack on which they were working.

Group problem solving

The same negative pressures were exerted against the problem solving sessions and, in the early part of the course, tutors found themselves having to insist that time be spent in this way. However, once the students observed the benefits of these sessions, attitudes changed. The Avery Hill students, more so than the Hackney students, were aware of a great deal of mathematical learning being accomplished through the discussion and questioning which was the mode of the problem solving sessions. The Hackney students were more committed to the individual use of the packs and consequently more resistant to the use of games, investigations, etc. At this stage, it is difficult to assess whether this is a function of the differences in goal of these students (that is the pressure to complete their access course in the nine available months), a difference in tutor, the effect of the pilot nature of the course, or some combination of these and other influences. However, for the coming academic year, more emphasis on group learning in the packs as well as in the problem solving component of the course will be made. The team is confident that this will reinforce course aims and objectives considerably as it was observed how fluent was the transfer of processes from problem solving sessions to pack work, for example in the use of different methods of recording.

Pack contents

Students requested the addition of a terminology sheet to each pack despite the free availability of mathematical dictionaries. They found difficulty in familiarising themselves with the necessary mathematical language when under time pressure. Marking sheets for pack materials for consultation by students

who were uncertain were also requested. One suggestion still under consideration is for the addition to the packs of a suitable textbook as an added resource. Given the background and history of these students, it is not clear what would constitute a 'suitable' text.

The packs were developed under great time pressure on the team and, as a result, often fell short of the ideal. In particular, activities were included which students identified as unhelpful or ambiguous and sometimes the sequencing of the material was discontinuous. However, these difficulties became apparent through the students' evaluations of the materials and their identification was, in itself, evidence of increasing confidence. Some test questions were criticised for ambiguity and lack of clarity, interesting in that most questions were taken from former public examination papers. Some test questions were poor reflections of pack contents. The major difficulty with pack materials was in finding a broad enough range of interesting activities which would support the slowest learning student. There was a tendency where extra activities were required to fall back on drill and practice exercises which were neither activity nor enquiry based. With the insistence by tutors on reliance on group work with games and puzzles, this problem should disappear. Another difficulty was incompatibility of software between Avery Hill College and Hackney, which meant that all the microcomputer investigations and games which were planned to support these students were unavailable to them. This was a particular blow with the geometry packs which were dependent on the use of LOGO. Again, it is hoped that this will not be the case for the coming academic year.

Tutor/Student Interaction

Most students commented positively on the classroom climate and its contrast with their earlier experience in learning mathematics. Some students, however, did note that it took them some time to adjust to self-responsibility instead of their expectation of a 'this is how you do it' approach from the tutor.

However, the tutor's role of facilitator was seen by many students as a positive part of the course and they found the relaxed, collaborative atmosphere lessened anxiety and helped build their confidence.

Student Attitudes

Students enjoyed the packs, the style of working and the atmosphere. They commented freely on the lack of threat, and the confidence created by having short-term goals which were achievable. They appreciated being able to work at their own pace with no feeling of competition although some felt that it took a little time to appreciate that the only person against whom one competed was oneself. They valued the opportunities for collaboration and commented on their own growth of understanding in comments such as 'I now understand things I have been doing for years without knowing why'. Failure did not seem to threaten these students as they felt it could easily be retrieved. The whole environment was described by the students in human and personal terms, contrasting sharply with their earlier 'academic' experience, and contributing positively to their developing confidence. One mature female at Avery Hill who has completed only two packs in the year has been heard outside the mathematics room delightedly announcing on more than one occasion 'I can do decimals now!' A final comment from one very happy customer: 'I loved it all. Could play with it all day. Would recommend the system to anybody whether they liked maths or not'.

Tutors' Comments

The original intention that the pack should be largely self-generating and require little tutor support was unrealistic. Tutors found, especially with the early packs when students were reluctant to ask for help, that they had to be very sensitive to student need for intervention and very supportive of group dynamics. Tutors commented on the difficulty of knowing when to intervene or offer assistance and when to leave the students to develop their own thinking. They felt that part of their role was in helping students to identify difficulties and recognise strategies for dealing with these and in this

respect the problem solving component was invaluable. Some students found the problem solving more challenging than the packs and were observed spending considerable time and interest on it. Absences and students arriving late or leaving early did not affect this kind of class and the environment was consequently relaxed and unpressured.

Because the tests were seen as self-assessments and were taken in the classroom, the students responded to them positively and their success at passing them, after the initial bad start already mentioned, supported this.

Tutors would benefit from time to reflect on the course and share experiences of their students during its progress. Unfortunately, this has been largely impracticable during the pilot year except at the regular team meetings to work on new materials during which some time has been given to reporting back.

Development Plans

It is clear from our evaluation that the objectives of the course to capitalise on the experience and motivation of the adult students to present mathematics as an enquiry-based activity, to build confidence and to change perspectives were largely achieved. The students report their experience and learning in very positive terms. Our development plans, therefore, are for consolidation and refinement with an intention, in another year, of offering the package generally.

1. Completion time for packs: A statement at the beginning of each pack 'Do not spend more than ..... hours on this pack' was found to be inappropriate. Instead, each new student will be provided with a Personal Progress Sheet which will include the following information:
   (a) starting date for each pack
   (b) a review date for each pack negotiated with the tutor
   (c) finish date for each pack
   (d) test results for each pack

2. Assessment over all: Students were asked to compile a file of all their work, including reading and problem solving but during the pilot year this file was not assessed. Consequently, there was no public record of the problems on which a student had worked. Hence, in future, the Personal Progress Sheet will include, on its reverse side, a requirement for each student to keep a problem solving diary so that both pack and problem solving information will be readily available and will constitute a record of the achievements of a student.

A decision has yet to be taken as to what proportion of completed packs can constitute a satisfactory pass as thirteen packs were found excessive by many students in a year. Only students who had previously attempted 'O' level work managed all thirteen.

It was also felt necessary to ensure a form of external moderation between the tutor and students on the Access course and Avery Hill College. In future, Leone Burton will undertake this role.

AID FOR PROBLEM SOLVING
DIAGNOSIS AND PRESCRIPTION IN ACTION AT YEAR 12

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INTRODUCTION

The acquisition of problem solving skill is a cumulative self-modifying process. For a highly structured and cumulative domain such as Mathematics, it involves both general cognitive processes and the knowledge structure that is the content of Mathematics (Glaser, 1984).

The first years of schooling provide the foundation for the structured mathematical knowledge of children and set the attitudes, beliefs and expectations about Mathematics and its applications, thereby having a profound effect on problem solving ability. As Wheatley and Wheatley (1984) state:

> *"early childhood is a critical time to build readiness for problem solving".*

It is our experience that leaving problem solving to the upper years is too late for many children. Damage has been done in the early years that seems to prevent children from even starting problems. As the National Assessment of Education Performance testing has found (Lindquist et al, 1983), the rhetoric of problem solving has yet to be translated into classroom practice. We would have to agree with Glaser (1984) when, in discussing how structures of knowledge are typically taught in school, argues that:

> *"little direct connection is made with thinking and problem solving in the course of learning cumulative domains of knowledge".*

In the early years, the common classroom practice of imitative teaching of operations followed by one step word problems is too narrow for problem solving. Yet what should be added? Should there even be separate problem solving topics in the early years? Or should problem solving at this level be part of the everyday teaching?

In reporting on our classroom research, we have tried to come to grips with the type of problem solving that is appropriate for Year 2 children, and how its component skills can be diagnosed and subsequent instruction prescribed. This paper is written to share what we have done and formulated to date.

BACKGROUND

Our classroom research is part of a problem solving project of the Learning Assistant Centre of Brisbane C.A.E. This centre was set up to assist remedial staff, teachers, parents and College students to overcome children's learned disabilities and to provide a vehicle for staff research (Irons and Jones, 1984).

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* Ideas for this paper have been derived in part from classroom research funded by the Queensland Board of Teacher Education. However, the views expressed herein do not necessarily reflect the policy of the Board of Teacher Education.

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The project has three aims:

to analyse, via clinical interviews and teaching experiments, children's problem solving processes, and their acquisition;

to review current research literature in cognitive psychology (artificial intelligence and information processing) and neuro-psychology (hemispherical lateralization);

and to develop, trial and evaluate instructional materials.

As a result of this, we hope to develop:

models of information processing and models of teaching and learning which will accommodate existing research findings and provide insight into children's processing;

a hierarchical categorisation of problem, learner and environmental characteristics that can form the basis of a taxonomy for diagnosing problem solving skill;

and an instructional framework that can form a basis for curriculum with emphasis on content and teaching strategies.

GUIDING BELIEFS and THEORETICAL FRAMEWORKS

Our view of problem solving is based on the constructivist epistimology (Hunting, 1980; Von Glaserfield, 1978), developed from the work of Kant, Hegel and Piaget, that sees knowledge as that which is constructed anew by each learner through the interaction of the human mind with its environment. We view children as goal seeking individuals, who, through processing information, structure their own knowledge. We believe that models of human processing emerging from modern research in information processing, artificial intelligence and cognitive science will provide insight into the teaching and learning. Along with Hunting (1980) we believe that:

> *"The challenge then, that is set before the mathematical education researcher, is to carefully and deliberately develop viable models of intellectual functioning with respect to the acquisition of knowledge ...."*

We believe (in agreement with Wilson, 1976; Ginsburg, 1981; Confrey, 1980; and Hunting, 1980) that the effective tools for this task are the clinical interview (Opper, 1977) and the teaching experiment (Kantowski, 1978).

Our view of problem solving, derived from Newell and Simon (1972), Meiring (1980) and Charles and Lester (1983), is one which perceives an initial state and a final goal state which the individual is interested in achieving but must overcome some blockage to do so. Our view is encapsulated by Wheatley and Wheatley (1984) who see problem solving as meaning:

"What do you do when you don't know what to do?"

We identify a continuum of problem types: namely routine to creative (Anderson, 1980). Routine problems are based on content only and are usually the more common type presented to primary school children, especially in the early years. They are designed to reinforce children's understanding of a particular operation as well as to provide practice in applying computational skills. While these problems do require some interpretation on the part of the child, they can be solved by the application of a previously learnt operation e.g.,

John had 85c to spend at the shop.
He bought a drink for 60c.
How much money did he have left?

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As we move along the continuum, problems incorporate both content and problem-solving processes. They are an extension of routine problems where the child must transfer and/or restructure previously acquired knowledge in order to solve the problem. Problems of this nature are usually presented to children in the upper primary years. At the end of the spectrum, creative problems focus solely on the processes of problem solving where the child is confronted with something he/she does not recognize and to which he/she cannot apply a previously taught algorithm or model for solution. The process of obtaining a solution is considered more important than the solution itself, and often there is more than one possible solution e.g.

Three boys are in front of Jane in a line.
Six boys are behind Jane.
How many children are in the line?

The theoretical framework for our research is a model of the human learner as an information processing system developed by Nason and Cooper (1984). A simplified version of the model appears in Figure 1:

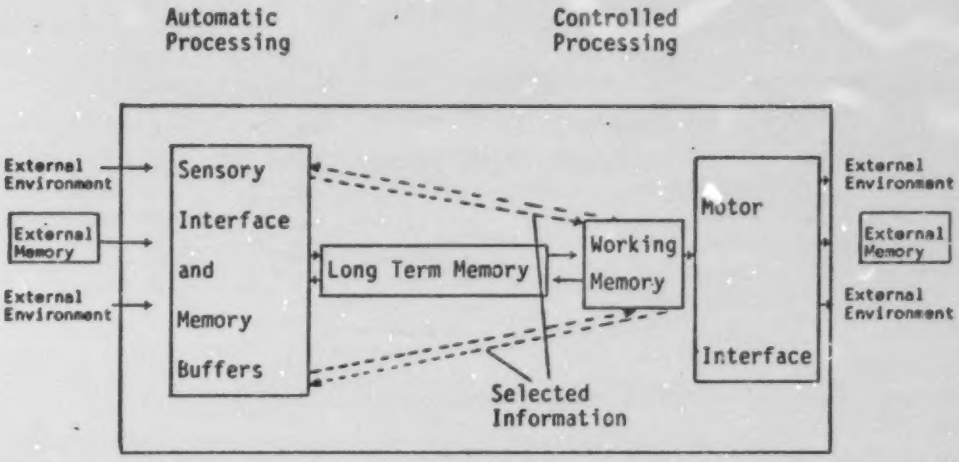

FIGURE 1

Information enters the system via the sensory interface where it is stored for a short time in the memory buffers. Here the information is processed automatically (unconsciously) in relation to the Long Term Memory and some information is selected for controlled (conscious) processing in the Working (or semantic short term) Memory. The Long Term Memory is where information is stored and is the source of internal information for processing. The information is stored as production rules (Newell and Simon, 1972; Young, 1976) in a network of nodes, within which it is possible to identify a continuum of three levels (Klahr and Wallace, 1976): specific, the basic content acquired directly from experience (e.g. the content structure of Lesh, 1981); to strategic, the self-regulatory ("housekeeping") and problem solving strategies which facilitate the processing of information and learning of new knowledge; to executor, the superordinate executive schemes (Case, 1974) which organise the application of all levels of knowledge. The Working Memory is where controlled processing occurs. It has severely limited capacity but is able to structure new knowledge in three ways (Anderson, 1982): coding (identification of a regularity); composition (replacing two or more production rules with one all-encompassing production rule); and proceduralization (removal of redundant conditions in a proceduralized production rule). The External Memory is a quickly accessible storage for information (e.g. words, numerals, drawings, doodlings, video, etc.) created by the Working Memory to overcome its capacity limitations. The Motor Interface operates the body and directs the senses, its most important function in this model being the medium by which the Working Memory controls output to the external environment and the External memory.

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The important implication of this model is that it allows the characteristics of problem, learner and environment that affect problem solving ability to be identified. At the same time it stresses the need for this type of analysis which allows problems to be matched to learners thereby enabling problem solving instruction to be a diagnostic-prescriptive cycle. The identified characteristics (adapted from Cooper and Nason, in Press) which affect problem-solving performance are categorised in the table (Figure 2) below.

Area	Characteristics that Affect Performance
the problem	• the position of the problem along the routine-creative continuum (i.e. the extent to which the solution of the problem requires domain specific knowledge or general strategies) • context in which the problem is set; language; presence of extraneous data • procedures and numbers to be used - size and complexity of numbers; number and complexity of operations (Bell et al, 1981)
the cognitive state of the learner	• quality and quantity of information stored in long term memory • the organisation of this stored knowledge • working memory capacity and extent of external memory strategies • cognitive style - the need for closure; degree of consistency needed; ability to switch methods; ability to see similarities and differences and to separate out extraneous data; ability to work in different sensory modes, particularly visual spatial modes of processing (Krutetskii, 1976); field dependence and independence
the affective state of the learner	• attitude, motivation, perseverance and self concept • ability to handle stress and anxiety, willingness to take risks • attribution to cause of success or failure
the environment	• the directed/undirected and open ended nature of instruction • amount of pressure, extent of support and encouragement • availability of materials and opportunity to use External Memory

FIGURE 2

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LITERATURE SURVEY

There is an abundance of articles that offer excellent suggestions for the teaching of problem solving in the early years (e.g. Bley and Thornton, 1982; Bruni, 1982; Burns, 1982; Moses, 1982; Nelson and Kirkpatrick, 1975; Spencer and Lester, 1981; Wheatley and Wheatley, 1984). However, while the impact of the work of Piaget cannot be understated, nor the contributions of Case (1978) or Collis (1983), there appears to be a dearth of modern research into the information processing of young children as they solve tasks presented as a problem. Furthermore, much of the existing research (e.g. Greeno, 1979; Hiebert et al, 1982; Nason, 1984) is on children's problem solving performance on one of the few basic operations.

As highlighted by Estes (1978), information processing models have grown largely from research conducted on normal human adults (e.g. Greeno, 1978; Newell and Simon, 1972) and, although this has culminated in comprehensive theories, the typical study contains:

> *"no systematic concepts or ideas bearing on the way in which the structures assumed might have taken form during the life history of a given individual".*

As Estes warns, considerable effort may have to be expended in translation and interpretation to relate research on cognitive development to the information processing approach. However, as the previous model (Nason and Cooper, 1984) exemplified, marriage of selected aspects of adult information-processing theories with aspects of Piagetian flavoured developmental theories is creating self-modifying models of learning and problem solving which have the potential to explicate information processing of young children (see particularly Case, 1978; Anderson, 1982, and the modern work of Klahr and Wallace).

Many research findings are not equivocal. For example, Burton (1984) argues that mathematical thinking is independent of the content area in which it is applied and that:

> *"an over-conscientious concentration on the content of Mathematics can obstruct the development of the kind of awareness on which mathematical thinking is based".*

While this may be only one aspect of her argument, it seems to exhibit a different approach from Glaser (1984), who, in reviewing research on problem solving in domains rich in content, argues that it is the development of schematic understanding of the domain specific knowledge that provides the basis for expertise in problem solving. The answer to this dilemma lies in the evolutionary nature of knowledge acquisition as exhibited in children and in modern artificial intelligence models. As Glaser (1984) states:

> *"the results of this newer research and theory force us to consider the teaching of thinking not only in terms of general processes but also in terms of knowledge structure - process interactions".*

We must return to the constructivism of Piaget. As Norman (1980) states:

> *"We are not just more than children, we are different ... But in the study of adult cognition there seems to be the implicit assumption that once we come to understand adults, children will simply be seen to be at various stages along the pathway towards the adult. Perhaps. But perhaps also that the complexity and experience of the adult will forever mask some properties ... Much of cognitive behaviour could be studied best through the developmental cycle, with the history of the development leading to better understanding of the adult".*

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THE RESEARCH PROGRAM

A research grant from the Queensland Board of Teacher Education was received by the Learning Assistance Centre to undertake classroom research in a local state primary school to develop diagnostic and instructional materials in problem solving in mathematics. One of our tasks, within this research, was to develop materials for Year 2 children. Because of the paucity of research information in this area of study, we found it necessary to restrict our research program to fifteen children and to study this group in detail using discovery oriented qualitative techniques (e.g. open-ended clinical interviews - See Ginsburg, 1981) to generate ideas and to develop models. Therefore we commenced a program of clinical interviews and teaching experiments as outlined below:

(1) observations of, and interviews with, children solving problems in informal teaching experiments to analyse children's ability to solve a variety of problems along the routine-creative continuum, to check their readiness for a variety of problem solving situations and to evaluate their reactions to discovery oriented open-ended instruction;

(2) surveys of the literature and development of learning and instructional models to provide frameworks for diagnostic instruments (these were developed from work in Years 4 and 6 also);

(3) preparation and administration of diagnostic instruments (clinical interviews and pen and paper tests) to evaluate children's readiness for problem solving and performance in selected aspects of problem solving;

(4) formal teaching experiments to analyse children's acquisition of new problem solving skill; and

(5) preparation and administration of summative diagnostic instruments (particularly clinical interviews) to ascertain the extent to which problem solving skills have changed.

We hoped to use the results from this program to develop and trial (in other classrooms) instructional materials in problem solving.

As our program is in progress at the time of writing this paper, we can only share with you the results of the informal teaching experiments and interviews, the model development and the clinical interviews. We will focus firstly on the models.

THE LEARNING MODEL: AID

During the course of the early informal teaching experiments conducted at Year Levels 4 to 6 as well as Year 2, we realized we needed to consider a number of aspects in order to meaningfully observe the children solving problems. It was necessary for us to analyse the skills they brought to the problem, the way they tackled it and what they learnt from their problem-solving attempts. For this reason, we developed a model (see Figure 3) which comprises three phases: approach, implementation and development (AID).

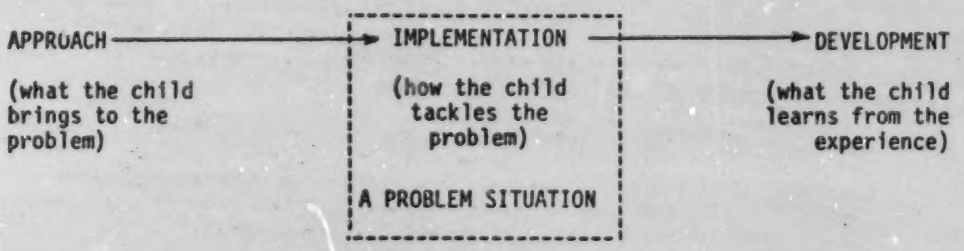

FIGURE 3

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The first phase, approach, focusses on what children actually bring to problem-solving situations. It is the starting point in considering the nature of children's behaviour in such settings and is related to the characteristics displayed in Figure 2. In a similar manner to Moses (1982), the model classifies these characteristics:

(1) affective traits

- attitude and personality factors such as motivation, perseverance, self-concept, preparedness to take risks, ability to handle stress and anxiety, willingness to take risks, etc.
- attribution to cause of success or failure
- working environment (experience in problem situations, capacity of present work situations to allow freedom required in problem solving, familiarity with problem solving situations, etc.)

(2) thinking abilities

- visual thinking (ability to use pictures and diagrams; extent of visual imagery, etc.)
- logical thinking (ability to see difference and similarity; to be consistent)
- patterning skills (ability to find and continue a pattern; to recognize attributes and see a pattern in their relationships)
- flexible thinking (ability to think flexibly, to change mind; to view from more than one perspective; to ignore extraneous data; ability to use a large number of sensory modes, etc.)
- creative thinking (ability to generate new instances to satisfy criteria; to think creatively; to put off closure, etc.) (See Burns, 1976, for excellent examples of creative thinking)
- Working Memory capacity

(3) knowledge

- reading and language abilities
- mathematics content (the extent of knowledge of facts and procedures, the organization of the content - the schematic understanding of concepts and processes, etc.)
- problem solving strategies
- executors

During the course of teaching the children the skills of solving word problems and detecting patterns, we realized that our observations had to focus on the children's ability to:-

1. Work out what to do,
2. Select a method of doing so, and
3. Implement the chosen strategy.

In the upper school we found many instances where the children's solution process broke down because they couldn't work out what to do and even if they could, they failed to follow the steps necessary to achieve a solution, particularly if this was a time-consuming process. There was also an inability to check one's answer. Consequently, the second phase of AID, namely implementation, came into being.

It is an extension of the stages of problem solving, first developed by Polya (1973) and expanded upon by Charles and Lester (1983), Green et al (1982) and Krulik and Rudnik (1980). (A similar three-step model is in Burton, 1980). It also augments the model of the general organization of the problem solver proposed by Newell and Simon (1972).

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Implementation comprises four stages as in Figure 4 below. Each stage is a continuum of two types of action: namely, basic to secondary.

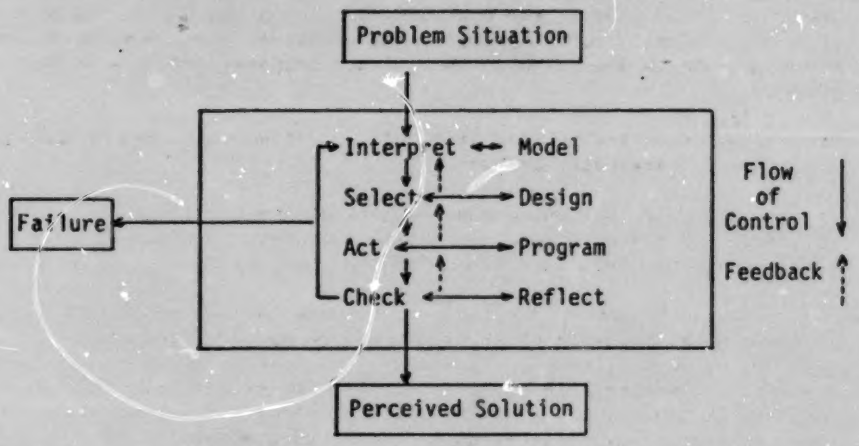

FIGURE 4

The basic actions are interpreting what the problem requires; selecting an appropriate method, acting on this method and checking to see if the solution obtained is satisfactory. The secondary actions, in which reside the more general problem solving strategies requiring time and effort, consume more Working Memory capacity and are only fired when the basic actions are inadequate. These secondary actions comprise:

- modelling which involves developing a model of the problem situation, together with the associated language and symbols (the interactions which occur here are discussed by Lesh, 1981, and appear in Figure 5 below);
- designing, where a plan of attack is devised before acting on the problem (see Anderson, 1980) and in which is included the ability to change model representation, as described in Newell and Simon (1972), and the ability to associate with a related problem, as described in Silver (1982);
- programming, where acting requires more than one step (e.g. iterative techniques); and
- reflecting, where the solution is subjected to scrutiny that leads to generalization and study of the solution process.

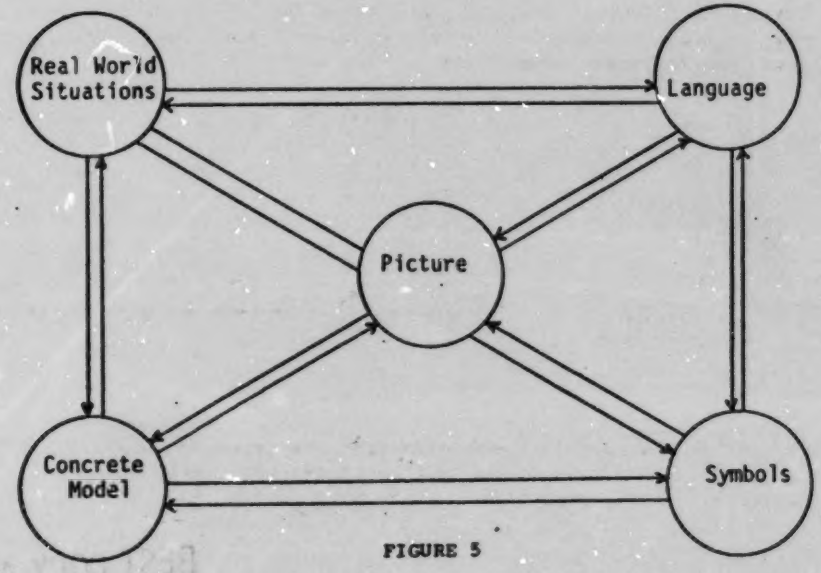

FIGURE 5

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The basic-secondary continuum is included to accommodate the research findings (see Glaser, 1984) which show that powerful content structure (or domain specific knowledge) facilitates problem solving in knowledge-rich domains. Hence, when routine problems are solved, the model operates at the basic action end of the continuum in each stage, providing a "smooth" path through the solution process. As problems become more creative, the model changes in operation to incorporate secondary actions in the solution process.

Of course, secondary actions need only be activated in one or some of the stages for a particular problem.

The linearity of the implementation part of the model enables it to be used to generate hierarchies and taxonomies, although it tends to deny simultaneous or concurrent processing, giving the model a naive appearance.

The third phase, namely development, focuses on children's capacity to grow and develop as a result of problem experiences. It is a process that is enhanced by success and by opportunity, motivation and Working Memory capacity to integrate the problem-solving experience as a learning. Development depends on the inter-relation between the problem solving and knowledge acquisition functions of the Working Memory. If the Working Memory is always filled to capacity with problem solving information, then little learning will occur. Hence, paradoxically, the more difficult the problem (for the child) the less the learning. Of course, we must remember that easy problems or those that are too routine also lead to little learning. Therefore, solving a problem which requires significant use of secondary actions in implementation (this may be caused by deficiencies in children's content structure, complexities in the problem or particularly, emphasis in instruction) may not result in improving problem solving skill unless the child is given time to reflect on the solution process. This concurs with some research findings that studying worked examples is a more efficient teaching strategy for developing expertise in problem solving than solving problems, e.g. Sweller, 1983, Sweller and Cooper, 1984. Furthermore, the reflection must allow for the structuring of information in areas useful to further problem solving. (This is reinforced by research findings which indicate that general processes are more effective for creative problems - refer, e.g. Charles and Lester, 1984.)

THE INSTRUCTIONAL MODEL: EMC

Having constructed AID to facilitate an analysis of problem-solving procedures and the learning required for expertise, we subsequently used it as a framework from which to develop instruction. The resultant instructional model has appeared effective in designing classroom instruction in problem solving. It evolved from teaching experiments using the AID model and is basically an adaptation of it. The three components of the instructional model, namely environment, methodology and curriculum are displayed in Figure 6.

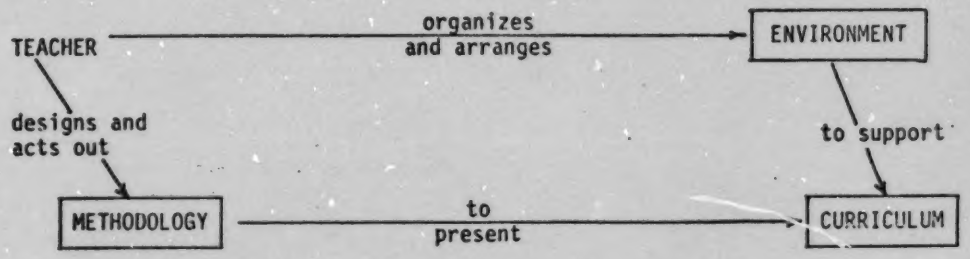

FIGURE 6

The components of this model have been discussed in Cooper (1983 a and b), Cooper and Nason (1983) and English (1984). For this paper we have only sufficient space to provide a list under each heading as on the following page.

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(1) *Environment*

- physical setting (attractiveness of room, chairs and desks available, etc.);
- resources (materials, consumeables, work cards, etc.);
- atmosphere (noise level, teacher support and encouragement, etc.); and
- classroom interaction (peer group relationships, amount of discussion encouraged, etc.).

(2) *Methodology*

- discovery orientation of instruction (extent of exploration permitted, open-ended questions posed, exemplars provided, etc.);
- emphasis on relational learning, i.e. development of schema (Skemp, 1978);
- diagnostic - prescriptive approach [matching of problems to children, techniques of reward and encouragement, marking schemes (see Charles, 1983) etc.]; and
- teacher commitment to problem solving (extent to which teacher takes risks in own endeavours - see Cooper, 1983).

(3) *Curriculum*

- General mathematics and language instruction [instruction to be in harmony with the approach to problem solving (see Wheatley, 1984)];
- readiness activities (extended from Charles and Lester, 1983) [activities designed to enhance motivation and positive self-concept (attitude and attribution), to develop visual or spatial thinking, logical thinking and patterning skills and flexible and creative thinking];
- process activities (activities to introduce procedures and frameworks that may assist children to develop strategies and executors);
- problem solving activities (solving problems, generalizing, discussing solutions, evaluating problem-solving capability, ensuring the development of efficient strategies and executors); and
- diagnostic and evaluative activities (activities to diagnose Mathematics ability, readiness, processes and problem-solving skill and to evaluate same).

Underpinning the EMC model is a belief that teaching problem solving in Mathematics is more than teaching a topic in Mathematics. In fact, it is seen as an extension of teaching Mathematics. As can be seen in the curriculum section, a problem solving program (as we view it) incorporates the general Mathematics program - adding to general Mathematics the additional requirements of teaching thinking skills, strategies and executors. One of the anticipated outcomes from our model building is that it will give insight into the teaching of these "additional requirements".

After analysing the AID and EMC models and reflecting on our research with children, we believe we have some preliminary proposals with regard to teaching thinking skills, strategies and executors. We would now like to share with you some of these initial proposals.

Our structure for teaching strategies (or, as we should say, the EMC structure for teaching strategies) is based on Figure 4, the implementation phase of AID. The strategies of Meiring (1980) have been assigned to the four stages of implementation (Figure 4) as in Figure 7 on the following page. Furthermore, we have formulated proposals for sequencing the teaching of these strategies and we also include these in Figure 7 (page 11).

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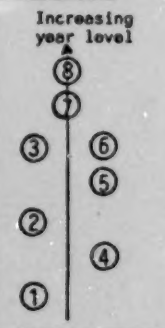
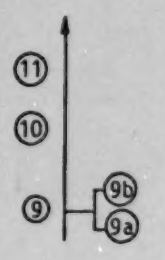
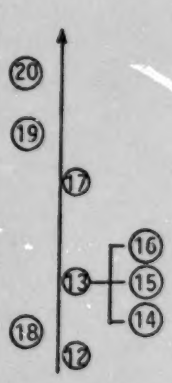
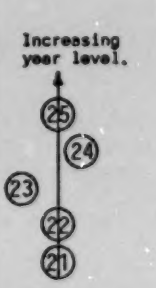
STRATEGIES SUITABLE FOR EACH STAGE OF IMPLEMENTATION (Those marked * defined as for Meiring.)	PROPOSED TEACHING SEQUENCE FOR THESE STRATEGIES
Interpret - Model Continuum 1. Re-read the problem. 2. Identify given, needed and wanted information. * 3. Restate the problem (in own words). * 4. Act it out. * 5. Make a model. * 6. Make a drawing, diagram or graph. * 7. Choose appropriate notation. * 8. Write an open sentence. *	*Interpret - Model Continuum* 
Select - Design Continuum 9. Solve a simpler problem. * 9a. Use simpler numbers. 9b. Simplify the conditions. 10. Think of a related problem. * 11. Change your point of view. * Check for hidden assumptions. *	
Act - Program Continuum 12. Identify and classify attributes. 13. Look for a pattern. * 14. Find and continue a pattern. 15. Construct a table. * 16. Systematically account for all possibilities. * 17. Think logically. * 18. Guess and check. * 19. Identify a sub-goal. * 20. Work backwards. *	
Check - Reflect Continuum 21. Check the solution. * 22. Find another solution. * 23. Find another method. * 24. Generalize. * 25. Study the solution process. *	

FIGURE 7

We do not have, as yet within our research program, hard evidence to support our proposals in Figure 7 on page 11. There is, however, support for our categories and teaching sequences in the interpret-model continuum from the work of Lesh (1981), particularly the modelling interactions described in Figure 5 (page 8). If, as in Figure 8 below, we add the appropriate strategies of Meiring (1980) to these interactions, we obtain an analysis similar to Figure 7.

THE LESH (1981) MODEL WITH STRATEGIES

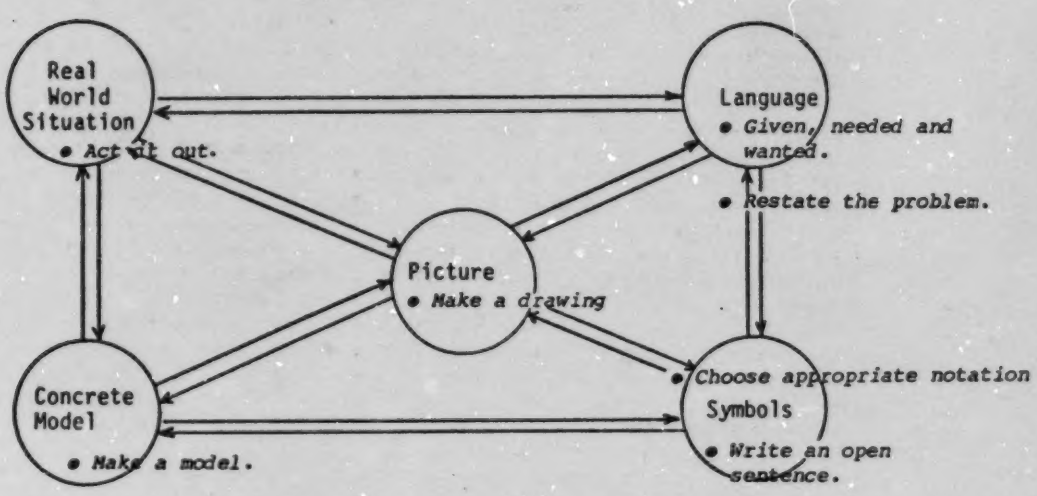


FIGURE 8

From an analysis of the AID and EMC models and the preliminary feedback from our research program, our proposals for the appropriate levels at which aspects of the EMC problem solving curriculum should be implemented in classrooms is as follows.

(1) The focus on thinking skills should begin in the early childhood years;

(2) The focus on the interpret-model continuum strategies should begin in the middle years along with the framework for implementation (in the hope that this will lead to formation of an overall "plan of attack" executor);

(3) The focus on act-program continuum strategies should begin in the upper primary years; and

(4) The focus on check-reflect and select-design continuum strategies should be left to the junior high school (although there are obviously some strategies in these two sections that should be focussed on earlier).

EXPERIMENTAL RESULTS

Although the development of the models was a major outcome of the classroom research, there were some particular results from the informal teaching experiments and the diagnostic testing that are worth sharing. These will have to be anecdotal because we have not yet quantitatively analysed our data (which is in the form of notes, diaries, reports, audio and videotapes). In fact, much of the model building has occured as a result of global conclusions drawn from discussion after observations and teaching experiments and has been a result of the process of recategorisation, necessary to develop the frameworks for effective observation (Ball, 1984). In many instances, the knowledge from which the models were developed, was initially personal or tacit (Polanyi, 1958).

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In the informal teaching experiments there were some interesting results in particular topic areas. In lessons focussing on logical thinking and patterning skills, the children were using attribute logic blocks to change a given sequence of blocks under direction such as, *"Change large blue to small red"*. They were also using unifix cubes to construct pattern sequences based on colour and number. Although there had been little previous instruction in both these experiences, the outcomes for each were very different. The children quickly and successfully progressed to quite complicated changes with the blocks but had difficulties even comprehending the idea of a pattern. The necessity to look along a sequence to get the answer instead of taking one thing at a time was difficult. In another lesson, children were constructing Lego vehicles and building from diagrams. This proved too great a jump for many students although they were highly motivated. Graded activities need to be developed using Duplo materials and copying actual constructions before copying drawings. Probably the most interesting was the sequence of lessons on subtraction problem situations. Most evident in these lessons was the adverse effect on problem solving of the children's limited prior knowledge of the meaning of subtraction. Great difficulty was had with situations which did not involve the basic take-away meaning of subtraction. (Here it would be interesting to investigate further the work of Vergnaud, 1982.) Most students were unable to create a list of possible subtraction and addition meanings from a given situation. As an example, a situation where there were two children outside a shop and three children inside the shop was enacted. Even with considerable encouragement, the children had great difficulty imagining the various subtraction and addition situations that could arise from this set-up. All of these experiences highlighted for us the importance of the readiness activities and the importance of knowledge in Year 2 problem solving.

From the models, the following diagnostic instruments were developed and administered.

(1) A pen and paper test of visual discrimination where the children were required to identify identical drawings and continue dot patterns.

(2) A pen and paper test to assess children's attitudes towards various components of mathematics. By colouring in faces ranging from happy to sad, the children indicated how they felt about such things as solving number sentences. By colouring in drawings marked ability, effort, task, or luck, the children disclosed the causes to which they attributed their scholastic successes and failures.

(3) A clinical interview schedule comprising the tasks shown in Figure 9 below.

TITLE OF TASK	DESCRIPTION OF TASK	APPROACH CHARACTERISTICS TESTED (See AID model)
Construction Task	• Copying a Lego construction • Completing a tangram exercise	Visual or spatial thinking Flexible thinking
Houses Task	• Working out which is Meg's house and who is Meg's father from statements about attributes • Digit span test	Logical thinking Working memory capacity
Counter Patterns Task	• Continuing pattern sequence using counters. (Simple patterns to complex patterns to patterns with extra material placed irregularly.)	Patterning skills Flexible thinking

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TITLE OF TASK	DESCRIPTION OF TASK	APPROACH CHARACTERISTICS TESTED (See AID model)
Clothing Cards Task	• Sorting attribute cards (different clothing, different colour, spotted, striped or plain) in as many ways as possible • Listing possible combinations of clothing that could be worn	Creative thinking
Word Problem Task	• Children represent, write and solve subtraction problems given in word, diagram and symbolic forms	Mathematics content structure Language
Probability Task	• Children interpret a drawing in terms of possible outcomes • Children invent stories for illustrated situations	Logical thinking Visual/spatial thinking Flexible thinking Creative thinking

FIGURE 9

Each of the tasks above had the capability of measuring more than the stated approach characteristics and commonly did so. Furthermore, overall impressions were important for conclusions about approach characteristics, particularly motivation.

On perusal of the results of the tests and interview schedule, the children showed remarkable diversity (strikingly so for attribution).

This was the case even for children who, in the teacher's opinion, performed equally well. This can be demonstrated by analysing the performances of two of the better students, Jill and Jack. (See Figure 10 below.)

DIAGNOSTIC INSTRUMENT	RESULT	
	JILL	JACK
Visual discrimination *(pen and paper test)*	*Could find drawing same as given one;* *Could not find two figures the same from amongst many.*	*Successful at both tasks.*
Patterns *(pen and paper test)*	*Successful at continuing pattern, but could not create a pattern when asked to do so.*	*Successful at all tasks and showed originality in creating own pattern.*
Attitude *(pen and paper test)*	*Marked everything "happy" except for "here's some easy work" (grumpy) and "keep quiet - work on your own" (no preference).*	*Much more diversity in markings. Grumpy face indicated for pictures, word problems and the statement, "here's some easy work for you".*
Attributions *(pen and paper test)*	*Tended to attribute cause of success to luck and failure to ability.*	*Attributed cause of success to ability and failure to effort and task.*
Interview Task 1 *(constructions)*	*Excellent at Lego and reasonable at tangrams though lacked flexibility.*	*Excellent at both and showed flexibility and creativity.*

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DIAGNOSTIC INSTRUMENT	RESULT	
	JILL	JACK
Interview Task 2 *(logic task and digit span)*	*Excellent at finding Meg's home and father.* *Very high digit span.*	*Excellent also, but did jump too quickly to a correct conclusion on only some of the characteristics.* *Only average digit span.*
Interview Task 3 *(counter patterns)*	*Continued patterns excellently in quite complex situations and had less difficulty than other children in detecting a pattern when materials were randomly added.*	*Good in all situations.*
Interview Task 4 *(clothing cards)*	*Could only see a narrow band of sorting criteria. Had to be prompted before she could see more than two possibilities.*	*Only saw some of the sorting options though realized immediately there was more than one way. Systematically formed all situations - best creativity shown of any of the children.*
Interview Task 5 *(subtraction word problems)*	*Strong interpretations of subtraction exercises.*	*Fairly complete basic facts and reasonable interpretations of subtraction exercises.*
Interview Task 6 *(probability picture)*	*Excellent answers of possible outcomes. Narrow interpretation of stories and inability to make up own story.*	*Excellent on outcomes, slightly more options in interpretation; tremendous ability to invent stories.*

FIGURE 10

Overall, Jack displayed far better attribution and visual, flexible and creative thinking. Jill showed slightly better logical thinking, aspects of patterning and basic skills. Her working memory capacity was considerably better than Jack's. He was familiar with many problem situations because his mother, as a trainee teacher, provides him with a variety of educational experiences. His rich milieu may be one of the reasons why he exhibits considerably stronger flexibility and creativity, although he appears to be less endowed in logic and working memory capacity. Jack may well be achieving at his potential. Jill's diagnosis is more straightforward: she requires readiness activities and severe bolstering of her self-concept.

FUTURE ACTIVITY

With a fairly comprehensive collection of models prepared, our future activity needs to focus on completing the research program, analysing the data as a test of models, and identifying any relationships that could point to more insights into the problem solving process. We would also like to use the findings and the models to develop, trial and evaluate diagnostic and instructional materials for classroom use.

In the more distant future, we would like to probe deeper within the models, looking particularly at:

(i) exploring affective traits, particularly attribution, and measuring their affect on problem-solving performance;

(ii) developing better materials and setting up more formal interviews and teaching experiments to investigate the relationship between affective traits, content structure and cognitive processes (strategies and executors) on the one hand, and routine and non-routine problems on the other;

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(iii) following through the developmental aspects of the acquisition of problem-solving skill and modifying the models to take into account developmental ideas; and

(iv) attempting to produce, using production systems (Newell and Simon, 1972; Young, 1976), computer simulations and cumulative models of information processing applicable to children in the early years of schooling.

* * *

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A SYSTEMATIC APPROACH TO DEVELOPING INSTRUCTIONAL MATERIALS FOCUSING ON MATHEMATICAL WORD PROBLEMS

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One of the major goals of mathematics instruction is to develop students' problem-solving abilities. In the early 1960s, the Cambridge Conference on School Mathematics (Educational Services Incorporated, 1963) urged curriculum developers to place much more emphasis upon problem solving in school mathematics. More recently, the National Council of Supervisors of Mathematics (1977) has defined problem-solving as one of the ten basic skills areas in school mathematics, and the National Council of Teachers of Mathematics (1980) has recommended that considerable effort be devoted to teaching students to read and solve word problems. However, students at all levels continue to experience difficulty with word problems. The latest NAEP assessment indicates there was very little change in student problem-solving performance between 1978 and 1982 (Carpenter, et al., 1980; NAEP, 1983). These results are disturbing and deserve serious attention. Further, many teachers become equally frustrated at their inability to help students develop problem-solving skills. These data seem to support the need to question the efficacy of current methods and materials for teaching students to read and solve word problems.

Effective integration of problem solving activities into the school mathematics curriculum has not occurred. In order to design instructional materials that facilitate the development of students' problem-solving abilities, the authors believe the approach to studying variables related to learning and instruction must be reexamined. Specifically, we need to learn some answers to critical questions such as: (1) What cognitive processes are involved in problem solving?, (2) Are some strategies more useful than others?, (3) Are there sex differences in students' ability to solve problems?, (4) What prerequisite skills are needed for efficient problem solving? (5) Can students' problem-solving performance be improved by instruction?, (6) Do teachers have adequate instructional techniques for teaching problem solving?, (7) Can usable diagnostic and prescriptive techniques for problem solving be developed?, (8) Can effective classroom procedures and materials be developed for teaching students to solve problems and easily integrated into the usual classroom situation?, and (9) Can effective inservice training programs focusing on techniques for teaching problem-solving be developed?

Information pertaining to the above issues cannot be obtained by traditional experimental studies alone. Uprichard (1984) suggests alternative methodologies which hold considerable promise for studying the complexities of problem solving. His research paradigm has six validation levels: theory, reality, clarification, refinement, intervention, and demonstration. The

paradigm begins with the articulation of a theoretical base or rationale (Level I) and moves progressively from exploration via naturalistic inquiry (Level II) to formulation of questions and hypotheses and hypotheses testing via clinical methods (Levels III, IV, and V) to confirmation or negation of statistical hypotheses via group experimental designs (Level VI). All six levels involve a type of verification of beliefs and patterns. The purpose and research method for each level is presented in Table 1.

The purposes of this paper are (1) to summarize a series of studies on problem-solving in relationship to the Uprichard paradigm and (2) to discuss instructional materials evolving from this research. Specifically, the research involved an analysis and synthesis of the current literature, the observation of children in classrooms, and administration of individual and group interviews. The desired outcome of these studies was the development and validation of effective instructional procedures and materials for teaching students to read and solve word problems. The authors believe that successive studies (a systematic approach) can provide useful information for altering our ability to deliver effective instruction which could significantly improve students' problem-solving performance.

SUMMARY OF SUCCESSIVE STUDIES

In order to develop a theoretical rationale for the work reported in this paper, a thorough, critical review of the

Table 1. Purposes and Research Methods for Levels of Paradigm

	Level	Purpose	Research Method
I.	Theory	Identification and Articulation of Theoretical Base or Rationale	Analytic-Synthetic Review
II.	Reality	Application of Theory to Real World	Ethnographic Methods
III.	Clarification	Formulation of General Questions Related to How Subjects Think	Non-structured Interview
IV.	Refinement	Development of Specific Hypotheses Related to How Subjects Think	Structured Interview
V.	Intervention	Development of an Instructional Design	Teaching Experiment
VI.	Demonstration	Confirmation or Negation of Statistical Hypotheses Relative to an Instructional Design	Group Experimental Designs

Excerpt from:

Uprichard, A. E. (1984) An alternative approach for studying mathematics learning/instruction. Focus on Learning Problems in Mathematics, 6, 1-10.

literature was conducted (Phillips, Uprichard, and Johnson, 1974; Phillips, 1982; Phillips, Uprichard, and Blair, 1983; Uprichard, Phillips, and Soriano, 1984). Major emphasis was given to studies dealing with word problems; however, these findings were viewed in the broader context of general problem-solving. Studies dealing with both psychological and pedagogical concerns were reviewed. Specifically, attention was given to the psychology of mathematics instruction, problem-solving models, and information processing theory (Beilin, 1975; Gagne, 1983; Dunlap and Knight, 1980; Kagan, 1963; Klausmeier and Goodwin, 1966; Lester, 1978, 1982; Newell and Simon, 1972; Polya, 1957, 1962; Schoenfeld, 1980; Webb, 1977; Yeotis and Hosticka, 1980; Wickelgren, 1974). Based upon this review and the authors' previous work on the identification of prerequisite problem-solving skills (mathematical and language), the Uprichard and Phillips conceptual schema for solving word problems was formulated (Phillips, Uprichard, and Johnson, 1974; Soriano and Phillips, 1982; Jackson and Phillips, 1983; Uprichard, Phillips, and Soriano, 1984).

The schema (see Figure 1) can be utilized to design viable tools for diagnosis (descriptive) and instructional sequences that facilitate the acquisition of abilities needed to solve mathematical word problems successfully (prescriptive). Embedded in the schema is a six-dimensional strategy for solving mathematical word problems. The dimensions identified are:

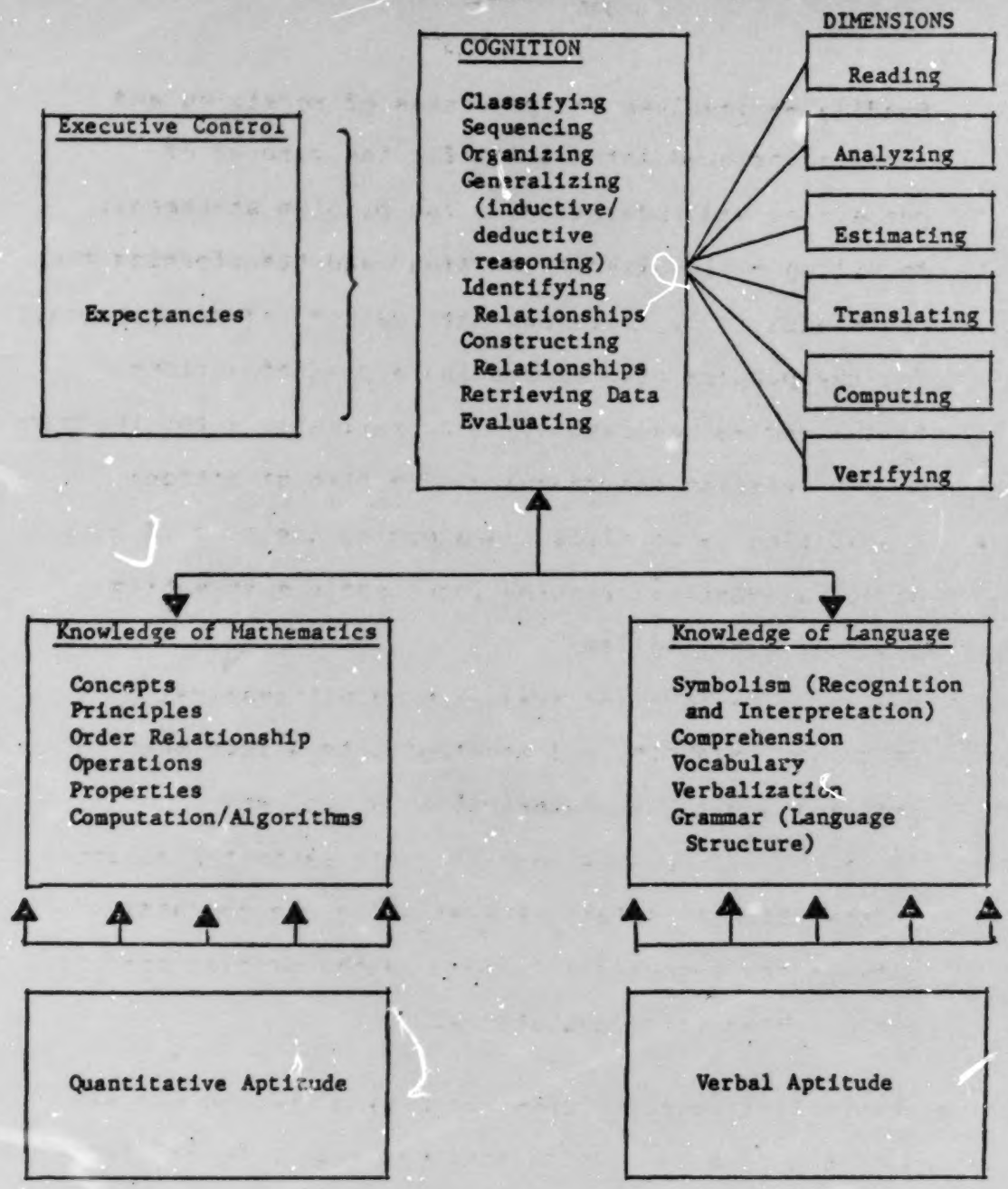

Figure 1. Conceptual Schema for Solving Mathematical Word Problems

Excerpt from:

Uprichard, A. E.; Phillips, E. R., and Soriano, A. (1984) A conceptual schema for solving mathematical word problems with implications for instruction. Focus on Learning Problems in Mathematics, 6, 79-107.

1. Reading -- involves the processes of receiving and encoding printed information for the purpose of perceiving and understanding the problem statement;
2. Analyzing -- involves organizing and transforming the input data (e.g., relevant information and problem goal) for the purpose of constructing a plan of action;
3. Estimating -- involves using approximations for the purpose of testing the adequacy of a plan of action;
4. Translating -- involves transforming the plan of action using relevant information into usable mathematical language or symbolism;
5. Computing -- involves transforming mathematical language, symbols, and meanings into a form that produces a possible numerical solution; and
6. Verifying -- involves comparing the estimated solution to the attained solution, evaluating the goodness-of-fit between the computed solution and the problem situation, and checking the computations.

The theoretical underpinnings for moving through the six dimensions of the problem-solving strategy relate to information processing theory. The authors believe that learning is an internal process which is affected by internal and external stimuli. In solving a mathematical word problem, a learner moves from an initial state to a goal state by creating a series of pathways that comprise a cognitive map. The learner creates this network of pathways by continually processing information related

to the problem context. The challenge in analyzing any problem-solving activity is in describing or prescribing the intermediate states (pathways) between the initial state and the goal state. It is suggested that the six dimensions of the problem-solving strategy presented in Figure 1 can be used to identify and/or categorize the intermediate states of problem-solving efforts.

In an effort to test the validity of the conceptual schema for solving word problems, observations of mathematics instruction were made in twenty-four first and fifth grade classrooms over a six-week period (Uprichard and Phillips, 1982). Records of both student and teacher activities relative to problem solving were written in a narrative form. The results of these observations were disappointing. Very little or no direct teacher instruction on problem-solving was observed. Further, students who were able to complete problems, presented in written form, moved directly from reading the problem to computing. In reading problems, children processed information directly as it appeared from left to right (Dudley, 1984). For example, if a smaller number appeared first in a subtraction problem, students usually added. In terms of the six dimensions, student behaviors were observed with reading and computing only. No useful information relative to the other dimensions of analysis, estimation, translation, and verification could be obtained. Thus, classroom observations were not sufficient to determine if a conflict existed between theory and reality.

To facilitate the categorization of children's problem-solving behaviors in relationship to the six dimensions of the Uprichard/Phillips problem-solving strategy, the non-structured interview protocol shown in Table 2 was developed (Uprichard and Phillips, 1982). The interview process was non-structured. That is, the interviewer deviated from the protocol when necessary to probe student responses in attempting to determine what and how the student was thinking. Further, it was necessary for the interviewer to ensure that the student demonstrated competency on each dimension of the problem-solving process before proceeding to the next in order to assess the student's performance on all six dimensions.

Nine students at both the first and fifth grade levels were interviewed. Students were of average ability; however, careful selection of subjects was not possible. Three interviewers conducted and taped the sessions, which were approximately 30 minutes in length. From this initial field testing, revisions in the interview protocol were made in terms of language and the sequencing of questions. For example, the order of estimation and translation were reversed. Translation led directly to computation and did not motivate or encourage students to estimate. The major outcome of this clarification process with a small sample was that students demonstrated no systematic approach to analysis, estimation, or verification. The only strategy mentioned by the students was "read, think and do." Unfortunately, students read and did (computed) without any thought to why they

Table 2. Non-Structured Interview*

I. READING

1. Symbol Recognition
   A. Read the problem aloud.

RESPONSE/OBSERVATION:

2. Comprehension
   A. Read the problem silently.

RESPONSE/OBSERVATION:

   B. Tell me the problem in your own words.

RESPONSE/OBSERVATION:

II. ANALYSIS

1. Important Facts
   A. What do we know?

RESPONSE/OBSERVATION:

   B. What do we want to find?

RESPONSE/OBSERVATION:

2. Plan (What has to be done to find the answer?)
   A. Action Steps

RESPONSE/OBSERVATION:

III. ESTIMATION

1. Evaluating the Plan (number sense)
   A. What do you think the answer will be close to:

RESPONSE/OBSERVATION:

IV. TRANSLATION (action steps incorporating mathematical language)

1. Organizing and Structuring Data
   A. Numbers/Computational Form
   B. Diagrams
   C. Tables
   D. Graphs

RESPONSE/OBSERVATION:

V. COMPUTATION RESPONSE/OBSERVATION:

VI. VERIFICATION

1. Product
   A. Is your answer correct?

RESPONSE/OBSERVATION:

2. Process
   A. Does your answer make sense? (must refer back to problem)

RESPONSE/OBSERVATION:

*Uprichard and Phillips, 1982

used a given operation or if the obtained solution made any sense in relation to the given problem situation.

The individually administered non-structured interview proved to be a highly useful tool for diagnosing a child's mathematical abilities. However, this technique is not practical for teachers to use in classroom diagnosis because of the time and effort involved. Thus, a structured interview involving the use of an audio tape and test booklet was developed (Uprichard, Phillips, and Soriano, 1983). The structured interview was designed such that the required administration time with a group is the same as that required for the non-structured interview with an individual. A basic concern is whether the non-structured and structured interviews yield comparable data for purposes of planning instruction.

Uprichard, Phillips, and Soriano (1983, 1984) assessed fourth and fifth grade students' performance on the six dimensions of the problem-solving process using both non-structured and structured interviews. Trained classroom teachers (ten) administered the structured interview to three hundred students. Each teacher also administered a second non-structured interview to a subset of four students (n=40). Interview data from students receiving both interviews was used to conduct a compatibility analysis. There was an 80% agreement between the two interview modes on both simple, single-operation and more difficult, multi-operation problems. An interdimensional error

analysis was completed on the structured interview data only. This analysis yields error ratios (ratio of incorrect responses to total responses) for each dimension of the problem-solving process. The greater the ratio the more difficulty students experienced on the dimension. The error analysis supported conclusions from previous work. Students demonstrated least competency with translation, estimation, analysis, and verification. The authors demonstrated that the structured interview can provide useful data for diagnostic and prescriptive teaching. Further, the structured interview can provide data useful in (1) better understanding how children learn to solve problems and (2) designing instructional materials that facilitate the development of children's problem-solving abilities.

INTERVENTION

The literature is replete with studies and models purporting the complexity of the problem-solving process. From a cognitive point of view, we need a better understanding of what processes are involved, how information is perceived, how strategies are developed, how information is stored and retrieved, how an individual selects and evaluates a previously learned strategy in a new situation, and much more. Answers to these questions come slowly. However, at present an unacceptable situation exists in mathematics instruction--students are not learning to read and solve problems. Obviously, improving this situation is a major concern of mathematics educators.

While much remains to be done, the development of instructional materials and procedures for improving student problem-solving performance is crucial. Based upon theory and the verification of that theory via classroom observations and interviews (non-structured and structured), the authors are developing a set of instructional materials. These materials, which directly reflect the conceptual schema, will be evaluated in the context of a teaching experiment. Presently, the instructional materials focus on the dimensions of the problem-solving process individually (prerequisites) and in combination (the six-dimensional strategy). No attempt is made to minimize reading or to state problems in the telegraphic format. This approach appears to be detrimental to the development of students' understanding of the problem situation (Moyer, et al., 1984). The materials being developed are language enriched. That is, both the prerequisite activities and the problems involve a considerable amount of reading; however, the language contributes to understanding the problem context and thereby motivates the student to attempt the exercises. Sample lessons (in draft form) with the skills and dimension being addressed by each are presented in Appendix A.

SUMMARY AND CONCLUSIONS

The studies dicussed in this paper represent a systematic approach to studying variables related to learning and instruction. Using the Uprichard paradigm as an information gathering

model, considerable data has been collected with respect to how students approach the problem-solving task and how teachers approach problem-solving instruction. The data is proving highly useful in (1) developing diagnostic and prescriptive techniques and (2) designing effective classroom procedures and materials for teaching students to read and solve word problems.

The systematic approach to developing instructional materials has been fruitful. The initial feedback from teachers and curriculum supervisors has been overwhelmingly positive and encouraging. However, the authors are not completely satisfied with their present attempts at implementing the data gathering approach via the Uprichard paradigm. Replications of these studies with more attention given to variable interactions need to be conducted and classroom intervention through carefully designed teaching experiments must be undertaken.

In summary, researchers and curriculum designers are far from completely understanding the problem-solving process. However, we believe that sufficient data is available to begin developing effective instructional materials for teaching students to solve word problems. Further, these materials and procedures can be easily integrated into the usual classroom setting and teachers can use them effectively with minimal training. In short, with improved instructional materials and more direct instruction (student-teacher interaction) in schools, student problem-solving performance can be improved.

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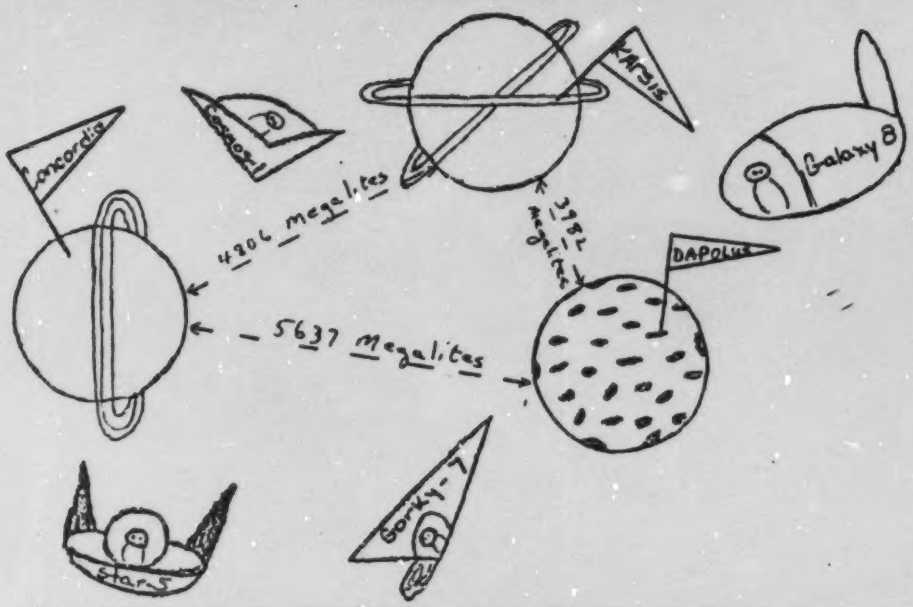
APPENDIX A

SAMPLE LESSONS FOR DEVELOPING PROBLEM-SOLVING SKILLS (Draft)

	Lesson	Dimension	Skills
1.	Space Travel	Read	Finding and using important information
2.	Chow Time	Read	Identifying the main idea and supporting facts
3.	The Flame Family	Estimation	Approximating
4.	Mountain Vacation	Estimation	Rounding and estimating solutions to problems
5.	Believe It or Not	Verification	Determining the reasonableness of statements
6.	Shopping Trip	All Six Dimensions	Integrating the six dimensions to read and solve problems

SPACE TRAVEL



Ship:	GALAXY-8	COSMOS-L	STAR-5	GORKY-7
Speed:	44 megalites	47 megalites	49 megalites	41 megalites per hour
Home Planet:	DAPOLUS	PLUTONIA	CONCORDIA	KARSIS

1. Which spaceship travels the fastest? _____
2. Which spaceship comes from CONCORDIA? _____
3. Which planet is not in the picture? _____
4. Which spaceship travels the slowest? _____
5. What is the home planet of GORKY-7? _____
6. Of the planets shown, which two planets are closest to one another?
   _____ _____
7. How many megalites is it from KARSIS to CONCORDIA? _____
8. Which spaceship would take the most time to travel from CONCORDIA to DAPOLUS?
   _____

Use data about Dapolians, Concordians, Karsinians, and the Plutonians to answer the following questions.

	D	C	K	P
Average height:	3 ft.	10 ft.	6 ft.	12 ft.
Average weight:	90 lbs.	100 lbs.	100 lbs.	200 lbs.
Number of arms:	3	4	6	2
Number of legs:	4	2	6	6
Population:	59 mil.	63 mil.	61 mil.	36 mil.

9. Which beings are the tallest? _____
10. Which planet has the greatest population? _____
11. Which beings have the most limbs? _____
12. Who would weigh less, a Dapolian or a Plutonian? _____
13. Which beings have the fewest number of arms? _____
14. Which beings have 3 arms and 4 legs? _____
15. Which beings have the same number of legs? _____
16. Which of the beings do you think would be the skinniest? _____

What do you think Plutonians look like? Draw a picture of one.

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What do you think Concordians look like? Draw a picture of one.

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CHOW TIME

Small dog | Medium dog | Large dog

Act

PIERRE | BUFFY | TINY

All dogs like to eat. Some owners feed their dogs too much. Others don't feed their dogs enough. The amount of food a dog needs to be healthy depends upon its size. Tiny, the brown and white St. Bernard, should be fed 12 cups of food once a day. Pierre, the white toy poodle, should be fed 1 cup of food four times a day. Buffy, the golden cocker spaniel, should be fed 4 cups of food twice a day.

DOG SIZES

Small	Medium	Large
Fox terrier	Cocker spaniel	German Shepherd
Toy poodle	Bassett hound	Collie
Chihuahua	Beagle	St. Bernard
Pekingese	Sheltie	Boxer

DIRECTIONS: Use the dog size chart and Chow Time paragraph to complete exercises 1 through 6.

1. Underline the main idea in the Chow Time paragraph.
2. Write three facts that give more information about the main idea.
   (1) _____
   (2) _____
   (3) _____

Circle the question(s) that best completes the problem.

3. Johnny has a German shepherd. Kim has a fox terrier. _____
   _____?
   (a) How much larger is Johnny's dog than Kim's?
   (b) How many more cups of food does Johnny's dog eat than Kim's?
   (c) How many cups of food does a medium size dog eat?

4. Sam has a cocker spaniel and a collie. _____?
   (a) How much does it cost Sam to feed the two dogs?
   (b) How many more cups of food does the collie eat than does the cocker spaniel?
   (c) How many cups of food do Sam's dogs eat each day?

Write two questions to complete the following problems.

5. Ed has a Chihuahua.
   (a) _____
   (b) _____

6. Ray has a boxer. Al has a beagle.
   (a) _____
   (b) _____

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THE FLAME FAMILY

FLICKER WICKER MRS. FLAME MR. FLAME

Mr. and Mrs. Flame had two children, Flicker and Wicker. Like all children, they were always comparing their heights to each other and to their parents. The sentences below are about the heights of the Flame family. Fill in the blanks.

1. Mr. Flame is about _____ times taller than Flicker.
2. Mrs. Flame and _____ are almost the same height.
3. Mrs. Flame is about _____ times taller than Wicker.
4. Mr. Flame is about _____ times taller than Wicker.
5. Flicker is almost _____ times taller than Wicker.
6. If Wicker stood on Flicker's head, they would be almost as tall as their _____.

Answer the following questions about the heights of the Flame family.

1. How tall is Mr. Flame? _____
2. How tall is Flicker Flame? _____
3. How tall is Wicker Flame? _____
4. How tall is Mrs. Flame? _____

Did you answer the questions? Why not? That's right, you need a unit of measure to find the height of each person in the Flame family. Answer the questions above by estimating the heights, using the Kandlelite stick. (Cut out the stick and measure the heights!)

1	2	3	4	5	6

KANDLELITE STICK

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Mountain Vacation

Mileage Chart

	Mt. Eagle	Stagecoach	Moose Jaw	Apache
Custer	518	140	98	214
Silver City	472	193	225	161
Bear's Paw	297	331	289	201

It is 225 miles from Silver City to Moose Jaw. Rounding to the nearest hundred, it is about 200 miles from Silver City to Moose Jaw. Rounding to the nearest ten, it is about 230 miles from Silver City to Moose Jaw.

Use the mileage chart to answer questions 1-4.

1. To the nearest hundred, how far is it from Bear's Paw to Stagecoach? _____

2. To the nearest ten, how far is it from Custer to Mt. Eagle? _____

3. It is about 100 miles from _____ to _____

   and from _____ to _____.

4. To the nearest ten, it is 190 miles from Stagecoach to _____.

Use the Road Map and the Mileage Chart to answer questions 5-8.

5. Josh and his father drove from Silver City to Bear's Paw to fish. To the nearest hundreds, how far did they drive?

   Silver City to Apache _____
   Apache to Bear's Paw _____
   Total _____

6. From the camp site in Stagecoach, the family drove through Silver City to Mt. Eagle to ski. About how many hundred miles did they drive?

   Stagecoach to Silver City _____ + Silver City to Mt. Eagle _____ = _____

7. Josh's sister drive from Stagecoach to Moose Jaw to shop. To the nearest tens about how many miles did she drive?

   Stagecoach to Custer _____ + Moose Jaw to Custer _____ = _____

8. How can you find out about how many miles it is from Custer to Bear's Paw?

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BELIEVE IT OR NOT

Tommy Tubb, a fifth grader, told his friends that he ate a 500-pound hamburger for dinner last night. If you were Tommy's friend, would you believe him? Can anyone eat a 500-pound hamburger? Does it make sense or seem reasonable? Write what you think!

_____

_____

_____

_____

_____

_____

_____

One way to tell if a statement or fact makes sense or not is to compare it with what we already know. We know that a hamburger is usually a quarter-pound or half-pound, at most one pound. We also know that after eating two or three hamburgers one is full. Eating a 500-pound hamburger just doesn't make sense or seem reasonable!

Below are some other statements that Tommy made. Circle each statement that doesn't make sense or seem reasonable.

1. My pet canary is 200 pounds.
2. I can throw a tennis ball over 15 yards.
3. I can run two miles in five minutes.
4. My $5.00 allowance is enough so that I can buy one candy bar each day of the week.
5. I can lift a 15-pound package.
6. I weigh 100 pounds, and my father weighs five times as much as I do.
7. I can run as fast as a horse.
8. I can swim underwater for two minutes without breathing.
9. I can throw a football over 50 yards.
10. I can drink as much as three gallons of water at one time.

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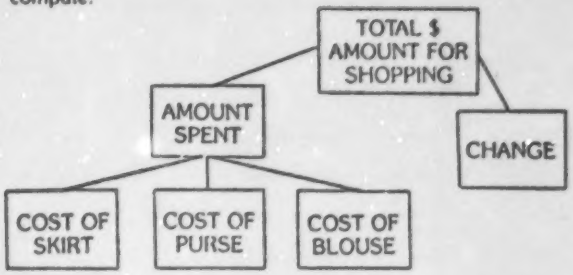
SHOPPING TRIP

Gale received $15 from her grandmother for her birthday. With what she got from her parents, Gale now has a total of $45 for shopping. If Gale buys a skirt for $14.85, a purse for $10.99, and a blouse for $11.98, what should be her change? (Don't worry about tax.)

1. Write the problem in your own words. Make it as simple as possible. Leave out the information that is not needed.

2. Fill in the diagram with all the information you can. Do not compute!



3. You know the total amount Gale has for shopping. If you knew the amount she spent, what operation would you use to find the change? ____
Complete the sentence:

Change = Total amount Gale has for shopping − ____

4. We need to know how much Gale spent altogether. Write the three facts that will help you find how much Gale spent in all.

5. What operation will you use to find the total amount spent?
____

6. Estimate the total amount Gale spent:
$14.85------ $15
$10.99------
$11.98------ _____ your estimate

Don't forget me
$

7. Compute the exact total amount spent:

8. Write your estimate here.---- ____
Write your answer here.------ ____
Compare them.
Is your answer reasonable? ____
Does your answer make sense? ____

9. Are you finished with the problem? ____
What is left to find? ____

10. Go back to the diagram and write the total amount Gale spent.
What box is still empty? ____
Now that you know how much she spent and you know the total, what can you do to find the change? ____

11. Complete the sentences:

Change = Total amount − Amount spent

□ = ____ − ____

12. Estimate the change Gale should receive:

$45 ---- $45
$37.92-- $38

your estimate

The answer is about $ ____

13. Look at these two number sentences. They will find the answer to the problem.

△ = _____ + _____ + _____
amount spent = cost of shirt + cost of purse + cost of blouse

□ = △
change = total amount − amount spent

Write the numbers you know in the correct spaces.

14. Compute the change: Total Amount ____
Amount Spent ____
Change

15. What was your estimate of the change? ____
What answer did you compute for the change? ____
Compare them.
Is your answer reasonable? ____
Put your answer back into the diagram.
Do all the numbers fit together? ____
Does your answer make sense? ____

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QUALITATIVE MATHEMATICS - THE FIRST PRIORITY

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Ontario
Canada

WHAT IS QUALITATIVE MATHEMATICS?

I first met the term "qualitative mathematics" some thirty years ago, when my mathematical education suffered some traumatic blows in the shape of Caleb Gattegno (1960) who figured prominently in the early development of the Association of Teachers of Mathematics in England. It was he who popularised the Cuisenaire rods. (They form a set of 10 rods each of cross-section 1sq cm. and varying in length from 1 to 10 cm.) One of his recommendations in using these rods was that initially the focus should be on how the rods related to each other e.g. the orange rod was longer than the red rod, the yellow rod was shorter than the blue rod. i.e. concentrating on relationships before quantity. The same was true if you "added" two rods together O + R = R + O etc so that it can be seen quickly not just that 3 + 4 = 4 + 3 but that 15 + 27 = 27 + 15. i.e. the generalisation of commutativity is obvious and applies to all pairs of numbers added together. This approach to mathematics became for me "qualitative mathematics."

APPROACHES TO MATHEMATICS TEACHING

Although there have been continuing attemps to change mathematics teaching in the last 30 years, especially in the U.S.A., Canada and the U.K., certainly it is my impression that in North America at least, we still focus essentially on teaching mathematics as a body of knowledge. i.e. we focus on transmitting to students a set of facts, formulae, well defined algorithms and theorems in arithmetic, geometry and algebra.

For example, we tend in North America to insist on well defined ways of adding, subtracting, multiplying and dividing numbers. Thus the decomposition method in subtraction is still almost universally used, with little emphasis on an alternative method such as adding on equal numbers. Thus

5 4	4'4	4'4	
-2 9	2 9	2 9	as opposed to
		2 5	
5 4	5 5	5 5	
2 9	3 0	3 0	which makes
		2 5	

subtraction a lot simpler, as it reduces the memory work, emphasising principles and relationships instead.

With the universal availability of calculators, and the rapid onslaught of the microcomputer, the need to do laborious written calculations is quickly disappearing - yet is being replaced by a corresponding need to understand how to enter calculations and how to be sure that the answer which emerges is the correct one for the calculation devised.

In other words, it is my thesis that we need to be searching for and emphasising ways of teaching and learning mathematics which focus less on the teaching of specific procedures, and memorising formulae and theorems, but which emphasise relationships, how things vary, and what the basic meaning of the ideas, and results are.

THE NATURE OF MATHEMATICS AND MATHEMATICAL THINKING

Questions about the nature of mathematics and mathematical thinking have occupied many people's attention in recent years, as we strive to improve the learning and teaching of mathematics in schools. Very briefly, mathematics as an area of study seems to have evolved in two ways. In terms of content, it has changed at its frontiers from the study of number and space in the real world to the study of axiomatic systems and abstract relationships in a mental world created by mathematicians themselves as they began to play and conjecture with the abstractions and generalisations they first found in the real world. e.g. the invention of new numbers such as $i = \sqrt{-1}$ and the theory of groups. Of course, these new systems have reality to other than mathematicians only when they can be seen to apply back in the real world. This is the necessary link between 'pure' and 'applied' mathematics.

The second way in which math has evolved has been the increasing study of how we actually do mathematics i.e. maths as a process. It may be said to have begun with Hadamard's book The Psychology of Mathematical Invention and gained greater impetus with Polya's work in Problem Solving in the 1950's and 1960's. It found expression in the discovery approach to teaching which surfaced in the 1960's, accompanied and followed by the activities approach in England of the Nuffield Project, and more recently of the Individually Guided Mathematics Project in Wisconsin, U.S.A. (Romberg and Montgomery, 1976). More systematic study of what are key activities in doing mathematics has resulted in the South Notts Math Project led by Alan Bell and others (1978). In

this project, four general strategies are emphasised. They are generalizing and abstracting; proving and explaining; representing and symbolizing; and writing up activities and results.

REFOCUSSING MATHEMATICS TEACHING AND LEARNING

I am not suggesting that it is unimportant for the student to have at his command a body of facts, basic skills, and well known formulae and theorems, - what I am suggesting is that more time and emphasis should be devoted to developing the meanings and relationships which go together to make up these specific summaries, generalisations and principles. Take the formula A=LB for example. What does this formula say? What does it mean? What restrictions apply to it? When is it useful? When is it useless? We should not simply be able to plug in numbers into the formula to gain answers to problems which are carefully supplied to us so that we use the formula unthinkingly.

Support for such re-focussing comes from a number of sources. Dienes (1959) writes clearly on the work of Piaget, who claims that learning occurs in cycles. First comes a play phase in which situations are explored without much constraint. This is followed by an increasing focus on some aspect which has meaning and structure. Some insight occurs and a new idea or relationship is found or discovered. Finally the understanding and integration of this insight has to take place through practice and consolidation. I <u>am</u> suggesting that more time and energy be spent in phases 1 and 2 so that the experiences which these represent can be richer and deeper.

Wheeler (1975) argues for the need to humanise mathematical education. He states his hope that "every child could experience at a few moments in his school career the power and excitement of mathematics" and suggests three ways of achieving this end.

1) by substituting mathematical activity for passing on mathematical knowledge.

2) by presenting the body of mathematical knowledge as it really developed in real life. To do so, he advocates taking as models the novel and the feature film rather than the textbook and the documentary.

3) use the idea of awareness - first enunciated some years earlier by Gattegno.

For Wheeler, awareness is like insight, but transcends it. It is "the kind of knowledge which comes to us as we concentrate on solving some problem or learning some idea or relationship". More specifically, it is "the act of attention that preserves the significant parts of experience". Wheeler cites as an example the awarenesses which are reached when we explore measuring an irregular shape to obtain its area. They include realizing that

1) we can refine the unit

2) we can repeat this process

3) anyone can make the measurement (i.e. the objectivity of area)

These awarenesses can then be used as tools in other situations.

Further support comes from the Cockcroft Committee Report (1982) and recent reflections by several authors. Plunkett (1981) suggests how what goes on in the mathematics classroom can be used to view and teach mathematics as a human activity involving various kinds of mathematical learning processes such as abstracting,

manipulating symbols meaningfully, being consistent in one's thinking etc., If this is the approach, then

> "one will try to introduce these activities to children. The aim will be to help them to experience them and enjoy them. The measure of success will be the extent to which learners subsquently think and act mathematically"
>
> (Plunkett, page 48)

Murray (1982) stresses a similar note in talking about her own experiences with the Rubik cube

> This I have learnt: the algorithm obstructs future learning. I find it very difficult even to try to understand how the middle layer works because I know the rule. And now looking back on the experiences of my own school-days, and of teaching in school, I really believe this to be the case. Rote learning seems to cripple the power to reflect, to play with an idea, to understand it and make it part of ones own 'mathematical reality'.
>
> (Murray, 1982 p 57)

Finally, Byers (1984), after analysing why his teaching did not seem to be producing the depth of understanding and insights into basic concepts he hoped for, concluded that he had been wrongly trying to be a good teacher by presenting his own pre-digested version of the subject as a way of shielding students from struggle and discouragement. He decided that this denied students the opportunity to venture into the unknown themselves. Since the teacher cannot learn for the student, his role is to create, yet

control the tension which results and encourage the student to learn to work productively with the teacher supporting him (or her) only when necessary.

TEACHING QUALITATIVELY

If it is accepted that students should be able to learn for themselves and that developing awarenesses is increasingly important in today's hi-tech world, then situations which contribute to these goals should be available for students to experience and explore. Among these situations should be some which have flexibility of structure and a degree of open-endedness.

Here are two illustrations of such situations. The first of these consists of a set of three investigations.

1. Choose any integer between 1 and 10, multiply it by the "magic number" 8547. Multiply the product by 13. Do this again with a different 1 digit number. Now try to explain your results.

2. Investigate how many prime numbers are of the form (a) n^2-1 (b) n^2-2 $(n>2)$. Justify your answers. What happens for n^2-3, n^2-4 etc.?

3. Investigate the advantages of a honeycomb being hexagonal rather than some other shape.

Clearly, these are at different levels of mathematical difficulty and intellectual development but they serve as indicators of the kinds of tasks which I believe are interesting and can be used intelligently and effectively by the teacher to foster individual thinking and collective discussion.

The second is the film M27 from the National Film Board of Canada Series "Experiencing Mathematics" developed by me with the technical expertise of the Board, particularly Sidney Goldsmith. For details of the film, the reader is referred to the series manual.

CONCLUSION

I trust that I have convinced you of the value of qualitative mathematics and suggested some sources and strategies which can help you examine again and re-focus your own teaching and that of others in what I am sure is a significant direction.

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DIFFERENCE, DIVIDING, AND MULTIPLICATION STRATEGIES IN TWO KINDS OF FRACTION PROBLEMS

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--ABSTRACT--

This study examined the use of processes in writing equivalent fractions. Subjects were 41 eighth-grade students in a Wisconsin (U.S.A.) junior high school. Two ten-item tests were given. In each, one fraction and the numerator (of denominator) of the second fraction were given; the student had to supply the missing denominator (or numerator). Mistakes (incorrect responses) were examined to determine error patterns. Three error strategies were identified: (a) maintaining the difference between the numerator and the denominator; (b) dividing the two given numerators (denominators); and (c) multiplying the quotient from the dividing strategy by the other given denominator (numerator).

The difference strategy seemed more common when the denominator was missing and the dividing and multiplication strategies seemed more common when the numerator was missing. Patterns of errors seemed fairly consistent for students who made few err[illegible]tudents who showed marked tendency for a particular strategy on o[illegible]e of the tests did not consistently use that strategy for the other test.

The primary purpose of this study was to try to infer use of processes in solving two kinds of fraction problems by examining the errors that students made in solving those problems. Of secondary interest was an examination of the consistency with which the same processes seemed to be used across the two problem types.

Background

In a recent survey of research on students' understanding of equivalence, Bright and Harvey (1981) noted that students typically have some difficulty in writing equivalent fractions. In part this may be attributable to the fact that "misunderstanding the individual objects that are to be made equivalent (i.e., common fractions . . .) may cause difficulty in understanding whether an equivalence exists" (p.156). The data surveyed on writing equivalent fractions involved missing-numerator problems only. This study extended those data to missing-denominator problems.

Searches of error patterns to infer processes students use to arrive at incorrect answers have often times restricted the domain of inquiry to the incorrect answers that students produce. Recently, however, with increased interest in computer simulations of human behavior arising from an information processing view of cognitive processes, there seems to be more interest in examining the entire range of responses a student produces. Bright (1984), for example, has pointed out that both correct and incorrect processes can produce correct answers, and in order to infer the

processes a student might be using, examination of all responses may be required.

Thus, a critical decision that must be made in error analysis is whether to examine only the incorrect responses or the full set of responses. The choice is certainly not automatic. Examining all responses may seem the most reasonable way to proceed, but if the percentage of correct responses is too high, there may be so much "noise" in the data that little guidance is provided for the teacher trying to remediate what may be only a few incorrect responses. Examining only the incorrect answers may provide insights that examining all the responses simply cannot provide. Both procedures will be used in this study.

For this paper, a distinction between "error" and "mistake" will be made (Bright, 1984). "A mistake is an incorrect answer given in response to a particular exercise (or problem). An error, on the other hand, is a consistent pattern of mistakes made in response to a series of similar exercises" (p.44). In this paper, there will be a search for errors by examination of the mistakes that students make in response to similar problems.

Procedures

Subjects

The subjects were 41 eighth-grade students in Wisconsin, U.S.A., who were participants in a co-instructional study of the use of mathematics instructional games (Bright, Harvey, & Wheeler, 1982, in press). These subjects had received some instruction on the writing of equivalent fractions, but mastery had not yet been achieved. The instructional game

was designed to supplement the ordinary classroom instruction on this important topic.

Tests

As part of the measurement of effects in that study, several pretests were given. Two of these were 10-item tests of writing equivalent fractions; one fraction and the denominator (numerator) of the second fraction were given and the student was instructed to write the missing numerator (denominator). The items are presented in Tables 1 and 2.

Analysis of Items

Based on the errors identified by Bright and Harvey (1981), the data were searched for three errors. Those errors are as follows:

1. Difference Strategy: The difference between the numerator and the denominator is maintained; e.g., 8/12 = 2/? becomes 8/12 = 2/6, with the difference between numerator and denominator kept at 4.
2. Dividing Strategy: The two given denominators (numerators) are divided, and the quotient is written as the required numerator (denominator); e.g., 8/12 = 2/? becomes 8/12 = 2/4, since 8 ÷ 2 = 4.
3. Multiplication Strategy: The dividing strategy is used, but is then followed by a multiplication of the computed quotient and the other given number; e.g., 8/12 = 2/? becomes 8/12 = 2/48, and since 8 ÷ 2 = 4 and 4 x 12 = 48.

The difference strategy is a primitive strategy that may be used more by less skilled students. The dividing strategy is a subskill of the

Table 1

Denominator-Missing Items

1.	$\frac{17}{14}$	=	$\frac{68}{\quad}$
2.	$\frac{3}{17}$	=	$\frac{9}{\quad}$
3.	$\frac{11}{2}$	=	$\frac{55}{\quad}$
4.	$\frac{3}{5}$	=	$\frac{9}{\quad}$
5.	$\frac{2}{6}$	=	$\frac{1}{\quad}$
6.	$\frac{13}{18}$	=	$\frac{52}{\quad}$
7.	$\frac{15}{11}$	=	$\frac{30}{\quad}$
8.	$\frac{5}{16}$	=	$\frac{25}{\quad}$
9.	$\frac{15}{14}$	=	$\frac{30}{\quad}$
10.	$\frac{8}{12}$	=	$\frac{2}{\quad}$

Table 2

Numerator-Missing Items

1.	$\frac{2}{18}$	=	$\frac{\quad}{9}$
2.	$\frac{3}{15}$	=	$\frac{\quad}{5}$
3.	$\frac{17}{7}$	=	$\frac{\quad}{21}$
4.	$\frac{6}{13}$	=	$\frac{\quad}{39}$
5.	$\frac{7}{19}$	=	$\frac{\quad}{57}$
6.	$\frac{9}{8}$	=	$\frac{\quad}{16}$
7.	$\frac{1}{16}$	=	$\frac{\quad}{32}$
8.	$\frac{5}{8}$	=	$\frac{\quad}{24}$
9.	$\frac{17}{5}$	=	$\frac{\quad}{10}$
10.	$\frac{18}{15}$	=	$\frac{\quad}{5}$

complete, correct strategy and may be a stage through which some students pass while learning that complete strategy.

The multiplication strategy produces the correct answer when the given fraction is in "lower terms" than is the to-be-written equivalent fraction. Since the pretests were written to measure achievement as a result of regular classroom instruction and were not designed primarily as diagnostic instruments, most of the items have correct answers which are consistent with the application of the multiplication strategy. Thus, it is not always possible to distinguish between correct processes and the inappropriate use of the multiplication strategy.

Tables 3 and 4 present the answers which are consistent with each of these three strategies. The correct answers are also presented, so that comparisons can be made across the processes being searched for. Note that for item 7 in Table 4, the dividing strategy and the multiplication strategy simultaneously produce the correct answer.

Results

The counts of the observed uses of the three strategies, along with counts of correct responses and blanks, are presented in Table 5. (Ten students gave only correct responses, so their data are omitted from further discussion.) In Table 6 these counts are adjusted by counting the use of the three strategies only for the incorrect responses.

Table 3

Predicted Answers: Denominator-Missing Items

Item	Strategy			Correct
	Difference	Dividing	Multiplication	Answer
1	65	4	56	56
2	23	3	51	51
3	46	5	10	10
4	11	3	15	15
5	5	2	12	3
6	57	4	72	72
7	26	2	22	22
8	36	5	80	80
9	29	2	28	28
10	6	4	48	3

Table 4

Predicted Answers: Numerator-Missing Items

Item	Strategy			Correct
	Difference	Dividing	Multiplication	Answer
1	X[a]	2	4	1
2	X	3	9	1
3	31	3	51	51
4	32	3	18	18
5	45	3	21	21
6	17	2	18	18
7	17	2	2	2
8	21	3	15	15
9	22	2	34	34
10	8	3	54	6

[a]X = strategy not applicable

Table 5

Process Counts

Student	Missing Denominator					Missing Numerator				
	Df[a]	Dv	M	C	B	Df	Dv	M	C	B
1	6	0	1	1	0	6	0	0	2	0
2	0	0	2	2	1	1	1	6	4	0
3	4	0	5	6	0	2	1	4	7	0
4	0	0	4	4	3	0	1	8	5	2
5	0	0	0	0	10	0	0	0	2	0
6	0	0	0	0	10	0	1	4	7	0
7	4	0	6	6	0	0	1	6	8	0
8	0	0	7	9	1	0	0	4	7	3
9	0	0	8	10	0	0	1	7	8	0
10	0	0	8	10	0	0	0	9	8	0
11	0	0	8	10	0	0	1	6	9	0
12	0	0	5	6	0	0	1	5	7	0
13	0	0	3	4	6	0	1	3	5	5
14	0	2	3	4	0	0	1	7	4	0
15	0	0	7	9	0	0	1	7	10	0
16	0	0	8	9	0	0	1	7	10	0
17	2	0	5	6	0	0	0	9	6	0
18	0	0	7	7	0	0	0	10	7	0
19	1	2	0	0	4	0	3	0	0	3
20	0	0	7	9	0	0	0	8	7	0

Table 5 (cont.)

Student	Missing Denominator					Missing Numerator				
	Df	Dv	M	C	B	Df	Dv	M	C	B
21	1	0	0	0	8	0	6	1	1	2
22	0	0	7	9	0	0	1	10	7	0
23	1	0	8	7	0	0	0	9	8	0
24	0	0	2	1	0	0	2	0	0	0
25	0	0	5	7	0	0	0	5	8	0
26	0	10	0	0	0	0	10	1	1	0
27	0	0	8	10	0	0	1	10	7	0
28	1	3	2	3	0	0	8	1	1	0
29	0	0	7	9	1	0	1	7	10	0
30	1	1	0	0	0	0	0	0	0	0
31	0	4	0	0	0	0	5	2	0	0
32	0	0	7	9	0	0	1	7	10	0
33	0	0	8	6	0	0	1	8	6	0

a Df = difference strategy

Dv = dividing strategy

M = multiplication strategy

C = correct

B = blank

Table 6

Adjusted Process Counts

Student	Missing Denominator					Missing Numerator				
	Df[a]	Dv	M	C	B	Df	Dv	M	C	B
1	6	0	0	1	0	6	0	0	2	0
2	0	0	0	2	1	1	1	2	4	0
3	4	0	0	6	0	2	0	0	7	0
4	0	0	0	4	3	0	0	3	5	2
5	0	0	0	0	10	0	0	0	2	0
6	0	0	0	0	10	0	0	0	7	0
7	4	0	0	6	0	0	0	0	8	0
8	0	0	0	9	1	0	0	0	7	3
9	0	0	0	10	0	0	0	1	8	0
10	0	0	0	10	0	0	0	2	8	0
11	0	0	0	10	0	0	0	0	9	0
12	1	0	0	6	0	0	0	0	7	0
13	0	0	0	4	6	0	0	0	5	5
14	0	2	0	4	0	0	0	3	4	0
15	0	0	0	9	0	0	0	0	10	0
16	0	0	0	9	0	0	0	0	10	0
17	2	0	0	6	0	0	0	3	6	0
18	0	0	0	7	0	0	0	3	7	0
19	1	2	0	0	4	0	3	0	0	3

Table 6 (cont.)

Student	Missing Denominator					Missing Numerator				
	Df	Dv	M	C	B	Df	Dv	M	C	B
20	0	0	0	9	0	0	0	2	7	0
21	1	0	0	0	8	0	5	0	1	2
22	0	0	0	9	0	0	0	1	9	0
23	1	0	1	7	0	0	0	2	8	0
24	0	0	1	1	0	0	2	0	0	0
25	0	0	0	7	0	0	0	0	8	0
26	0	10	0	0	0	0	9	0	1	0
27	0	0	0	10	0	0	0	3	7	0
28	1	3	0	3	0	0	7	0	1	0
29	0	0	0	9	1	0	0	0	10	0
30	1	1	0	0	0	0	0	0	0	0
31	0	4	0	0	0	0	4	2	0	0
32	0	0	0	9	0	0	0	0	10	0
33	0	0	2	6	0	0	0	2	6	0

a Df = difference strategy

Dv = dividing strategy

M = multiplication strategy

C = correct

B = blank

Several patterns emerge from examination of Table 5. First, the difference strategy was observed more frequently when the denominator was missing than when the numerator was missing. Perhaps students have less experience with a missing denominator, so they revert to a more primitive strategy in this unfamiliar situation. Second, the dividing strategy seems to have been used more frequently when the numerator was missing. However, the instances of a single use of this strategy are in almost all cases accounted for by item 7, for which the dividing and multiplication strategies produced the correct answer.

Third, for most of the students, the overall patterns of errors seem to be quite similar. The principal exceptions seem to be students 17, 21 and 28. Student 17 seemed to shift from a weak difference strategy on the missing-denominator items to an exclusive multiplication strategy on the missing-numerator items; students 21 and 28 seemed to shift to a dividing strategy for the second test.

The shifts between strategies are perhaps more clear from the data of Table 6. There is not a clear pattern, for example, in students who appeared to use the difference or dividing strategy four or more times on either test. Students 1, 3 and 7 used the difference strategy on the denominator-missing test, but only student 1 used this strategy on the numerator-missing test. Students 26 and 31 used the dividing strategy on the denominator-missing test, but students 21, 26, 28 and 31 used it on the numerator-missing test.

Analysis of the multiplication strategy in Table 6 is more sensitive, since the multiplication strategy could be used incorrectly only twice on the denominator-missing test and three times on the numerator-missing test. Of the 66 and 99 opportunities for use of the strategy, it was

observed 4 (6%) and 29 (29%) times, by 3 (9%) and 13 (39%) of the students. These data seem to support a difference in strategy use across the two tests.

Conclusions

The data of Tables 5 and 6 seem to suggest that students do not approach the problems in the two tests in the same ways, in spite of the mathematical similarity in the processes teachers may expect students to use. As suggested earlier, this may in part be due to the relative unfamiliarity of the denominator-missing format. However, it is just as reasonable to believe that students do not process the information contained in the numerator and the denominator of a fraction in the same ways. The instruction that these students were exposed to apparently did not communicate the structural similarity of the two situations. Without that recognition, students are not likely to apply similar processes.

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RESEARCH ON LEARNING AND INSTRUCTION: VARIABLES AND METHODS

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Learning and instruction phenomena have intrigued and challenged educational researchers for years. Discovering how a person thinks or learns and using that knowledge to design instruction is at the crux of the educational enterprise. However, accomplishing this is no simple matter. Fisher and Berliner (1979) have identified three characteristics which make the study of both learning and instruction a challenge. First, both are complex in that they are affected by an array of variables. Second, they are dynamic in that the variables affecting them are constantly interacting and changing. And third, they are extensive in time. That is, cognitive development takes place over extended periods of time, not an hour or a day. It is clear that learning and instruction occur in contexts that are not easily controlled.

In recent years, the potential for gaining new insights about learning and instruction has been enhanced by a shift in research emphasis from primarily a method orientation to a problem orientation. The method orientation is characterized by statistical design and generalization; the problem orientation by description and interpretation. With this shift in emphasis the researcher is no longer constrained by a particular statistical

model but is free to focus on what is most important, the problem. The position reflected in this paper is that future research initiatives on learning and instruction will be of most worth to the education community if grounded in general systems theory and information-processing theory, and employ multiple research methods.

General Systems Theory

A brief overview of general systems theory is necessary to help clarify what follows. To begin, the purpose of general systems theory is to bring about order and synthesis to complex and dynamic events so that conceptual relationships or connections between phenomena can be established (Berrien, 1968). Organizational schemas or systems are designed for use in identifying and describing variables and variable interactions. Specifically, a system is defined as a set of related elements or components. Attributes, which are either quantitylike or qualitylike, are used to describe components (van Gigch, 1978). Quantitylike attributes are well defined and measurable. Qualitylike attributes tend to be ambiguous or fuzzy and thus difficult to define and measure. When the total structure of a component is fuzzy and not well defined, it is referred to in systems theory as a "black box". In some situations one component can be considered a system or subsystem.

Theoretically, a system can be either closed or open. A closed system is one that is self-contained and accepts no input.

An open system is influenced or relates to other systems and accepts inputs. The flow of inputs into an open system is determined by its boundary or nature of its components. Component interactions transform inputs to outputs or products. Outputs at one level of a system can be defined in terms of attributes and considered as inputs at the next higher level in the system.

The three basic functions of an open system are description, explanation, and prediction. Only the predictive function is probabilistic in nature. However, the accuracy or power of this function depends in great measure on the specificity of the descriptive and explanatory functions. In behavioral science research the predictive function can yield two kinds of generalized statements: type and cross level (Miller, 1965). Type statements pertain to uniformities across individuals of a given class. Cross level statements pertain to uniformities extending across classes. If the generalized statements produced by a system are not useful or do not reflect reality then the system is suspect and requires modification.

Finally, one's work in a system is generally guided by a paradigm. A paradigm can be thought of as a set of procedures or methods, not necessarily sequential, that can be used in seeking solutions to given types of problems.

Table 1 contains important terms from the previous discussion on general systems theory.

Table 1. Selected Terms from General Systems Theory

System

Components

Component Attributes

- o Quantitylike
- o Qualitylike (ambiguous/fuzzy)

Black Box Concept

Closed System

Open System

Inputs - Boundary

Outputs - Component Interactions

Levels (of System)

System Functions

- o Description
- o Explanation
- o Prediction

Generalized Statements

- o Type
- o Cross Level

Paradigm - Procedure(s)/Method(s)

An Application of General Systems Theory to Research on Learning and Instruction

The System

A system focusing on behavioral science phenomena is an open system; one that is influenced by the environment and other systems. Not to recognize this fact would cause difficulties in evaluating the usefulness of such a system's products. However, the subsequent discussion is limited to a description of a single system. How the system might be affected by the environment is addressed later.

In designing a system it is necessary to do four things; state the system's purpose(s) or objective(s), identify its components, define each component in terms of attributes, and describe "holistically" how the components relate to or interact with one another.

The purpose of the system to be described is to facilitate researchers' efforts in gaining insights into how a person thinks or learns, and then using those insights to design instruction. Presently, four components comprise the system: learner, task, process and instruction. It is assumed that most variables directly affecting learning and instruction are classifiable within one of these four components. As each component is briefly described, keep in mind that working with any system will bring about changes and refinements. Any listing of component attributes is by no means inclusive.

In this particular system, learner attributes or variables are identified as organismic, trait, state and entry knowledge (see Table 2). Many attributes could be identified and listed in each of these four categories. Some of them would be quantitylike, others qualitylike or fuzzy. Variables such as sex, age and aspects of physical disabilities and entry knowledge are quantitylike. Examples of qualitylike variables are intelligence, attitude and anxiety. Although qualitylike variables are difficult to precisely define and measure because of their ambiguity, some of these constructs have been refined to the point where they are amenable to measurement. For example, there are a number of instruments available to measure individual intelligence, each based on a different interpretation of the construct (Guilford, 1967, Terman and Merrill, 1960, Wechsler, 1958). Measurement of attributes is important since attributes are used for descriptive purposes. In this case, they are used to describe the learner and the flow of learner inputs.

The second component in this sytem, task, is the component with which educators probably feel most comfortable. The categories used to classify task attributes are content, logical, physical, and linguistic (see Table 3). Obviously, each of these categories can be subdivided. In school mathematics, content can be thought of in terms of number, measurement, and geometry. In

Table 2. Learner Attributes

- Organismic
  - Sex
  - Age
  - Physical/Neurological Abilities
  -
  -
- Trait
  - Intelligence
  - Attitudes
  - Anxiety
  -
  -
- State
  - Intelligence
  - Attitudes
  - Anxiety
  -
  -
- Entry Knowledge
  -
  -

Table 3. Task Attributes

- Content
  -
  -
  -
- Logical
  -
  -
  -
- Physical
  - Stimulus Conditions
  - Response Conditions
  -
- Linguistic
  - Syntax
  - Semantics
  - Symbiotics
  - Rhetoric

diagnosing mathematical difficulties, educators have traditionally used mathematics taxonomies to precisely classify content (Underhill, Uprichard and Heddens, 1981). For example, numbers can be classified as wholes, non-negative rationals, integers, and/or rationals. Within each of these sets we can further classify content as number concepts, relations among numbers, notation, meanings of operations, properties, computational skills, and problem solving.

Tasks can also be analyzed or described in terms of their logical structure. That is, the number and types of steps necessary in seeking a solution to a given task can be determined. Logical analysis can also involve identifying prerequisites. Physical attributes of tasks can be defined in terms of stimulus and response conditions and linguistic attributes in terms of syntax (form), semantics (sentence or word meaning or intent), symbiotics (signs and symbols) and/or rhetoric (change in message).

Comfort with task attributes is a result of confidence in definitions and measurement procedures. However, caution must be exercised. Some ambiguity or fuzziness still exists with respect to classifying logical, physical, and linguistic attributes.

The third component in this system, process, is the "black box" of the system. It is the component about which ore can be least definitive. Its structural complexity is such that it must be considered a system or subsystem itself. This writer believes

that information-processing theory provides the best context to most accurately describe this component.

Information-processing theory or "modern cognitive learning theory proposes that learning and remembering are brought about by internal processes that are affected by the external organization of stimuli, by executive control processes brought to bear by the learner, and by the contents of the learner's memory." (Gagne, 1983, pg. 10) Specifically, a learner receives stimuli from the environment. These stimulus inputs are transformed to information through processes such as encoding, recall, searching and retrieval, and responding (see Table 4). The specific encoding or recall scheme a learner chooses to use to process stimuli and/or information is influenced by his or her executive control processes; what he or she attends to by attending mechanisms. Little is known about these control processes except that learner characteristics or inputs, especially previously learned knowledge, impact them. Defining the processes that are used to transform information and how they work, and getting learners to think about how they think (metacognition) is the key to understanding learning. In this sense, learning research is a study of dynamic interactions.

While information-processing theory dominates the process component, other points of view with respect to learning or mental processes (psychometric and developmental) influence the system through the learner component.

Table 4. Process Attributes

- Executive Control
  - Encoding
    - Strategies/Rules
  - Recall (Immediate, Delayed)
  - Searching and Retrieval
  - Response Generation
  - Evaluation/Monitoring
  -
- Attending Mechanisms
  - Focus
  - Purpose
  - Evaluation/Monitoring
  -

The instructional component is the fourth and final component of the system. Although described last, it is by no means any less important than the other three. Examples of instructional variables are mode of presentation (stimuli), response mode, sequencing, practice, time/pacing, reinforcement, etc. (see Table 5). Manipulating these types of attributes or variables so that learning processes are operating at a maximum to bring about desired outcomes is what instruction is all about. Unfortunately, knowing what variables to manipulate and to what degree is not always clear because of limited knowledge about learning processes. On a positive note, instructional variables are mostly quantitylike and can be operationally defined and measured. Once more is known about learning processes the actual manipulation of these types of variables should not prove to be a major problem.

The previous discussion has provided a brief description and skeletal outline of the system's four components. An appropriate question at this time is "How do the components fit together or relate to one another?". An analysis of the algorithm presented in Figure 1 helps to clarify these relationships. The two steps of this algorithmic process parallel the purpose of the system. The focus in Step 1 is on discovering how a person thinks or learns. Learner inputs interact with task inputs to yield process outputs. The primary concern in Step 2 is on designing instruction. Process outputs from the previous step are considered as

Table 5. Instructional Attributes

- Mode of Presentation (Stimuli)
- Response Modes
- Sequencing
- Practice
- Time/Pacing
- Reinforcement
- Examples/Non-Examples
- Advanced Organizers
- Questioning Techniques
-
-
-

[Learner (I) Task ⇒ Process] (I) Instruction ⇒ Instructional Design

or

Step 1. Learner (I) Tasks ⇒ Process

Step 2. Process (I) Instruction ⇒ Instructional Design

Figure 1. Algorithm

inputs at this level and interact with instructional inputs to yield an instructional design. There is high interdependency between components.

Step 1 of the algorithm is diagnostic and step 2 is prescriptive. Both steps involve the direct observation, description and interpretation of component interactions. In this context, the diagnostic/prescriptive process is defined as a dynamic science. This definition has important implications. For example, error or pattern analysis would not be considered diagnosis since no direct observation and description of learner-task interactions occur. In the future, more thought needs to be given to precisely defining what educational diagnosis and prescription is and is not. Current definitions are too loose.

In generating an algorithm, understandably the first concern is with relating comonents to each other in a manner consistent with the system's purpose or objective. A second concern relates to power or accuracy. In the particular algorithm being discussed, power or accuracy is assumed to be a function of specificity. That is, the probability of making valid type or cross level generalizations (predictions) about outputs or products increases as descriptions of input flows and interactions become more specific. In Figures 2a and 2b the specificity function is depicted graphically with respect to Step 1 of the algorithm. Higher specificity of component input flows more accurately focuses component interactions. Clearly, interpretation and prediction are grounded in description.

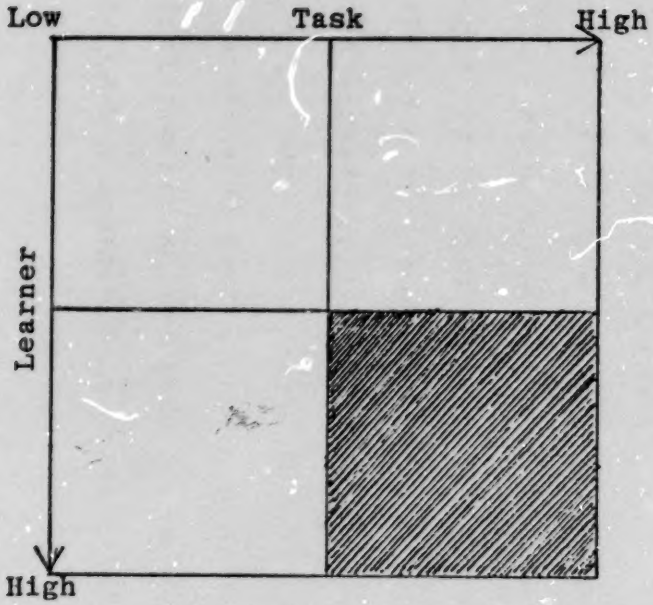

Figure 2a. Specificity Function - Step 1

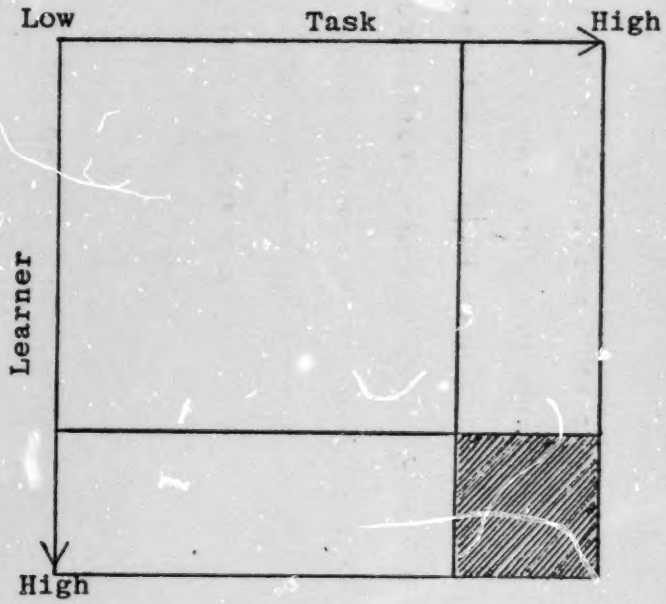

Figure 2b. Specificity Function - Step 2

To summarize, the purpose of this system is to facilitate researchers' efforts in gaining insights into how a person thinks or learns, and then using those insights to design instruction. The four components of the system are learner, task, process, and instruction. Component relationships are expressed or represented by a two-step diagnostic/prescriptive algorithm. Finally, power or accuracy within the algorithm is a function of specificity.

The Paradigm

Work in a system is usually guided by a paradigm. A paradigm can be thought of as a set of procedures or methods, not necessarily sequential, that can be used in seeking solutions to given types of problems. In a system designed to study behavioral science phenomena (learning and instruction) the paradigm would consist of research methods and procedures. The following discussion focuses on selected research methods and their relationship to the two-step algorithm presented above.

Previous exploration with research methods by this writer (Uprichard, 1984) has led to the development of a paradigm that has six validation levels: theory, reality, clarification, refinement, intervention, and demonstration. The paradigm begins with the articulation of a theoretical base or rationale (Level I) and moves progressively from exploration via naturalistic inquiry (Level II) to formulation of questions and hypotheses and

hypotheses testing via clinical methods (Levels III, IV, and V) to confirmation or negation of statistical hypotheses via group experimental designs (Level VI). All six levels involve a type of verification of beliefs and patterns. The purpose and research method for each level is present in Table 6. While the levels or validation steps are conceptualized as being sequential, it is possible to combine a research method at one level with methods from other levels or to alter the sequence of the levels to achieve a better fit between methods and problems.

Relationships between paradigm levels and the system's two-step diagnostic/prescriptive algorithm are diagrammed in Figure 3. Theory type activity serves as readiness for entry into the algorithm. Research at the reality level involves observation of both learning (Step 1) and instruction (Step 2). Research methods associated with the clarification and refinement levels are used to better focus efforts in gaining insights about how a person thinks or learns (Step 1). Research methods at the intervention and demonstration levels are used to focus on instructional design (Step 2). In addition to readiness, theory (Level I) spans across both algorithmic steps. While theory is important in that it establishes the initial research context, it will be modified by the research findings from work in subsequent paradigm levels. To facilitate understanding, a brief description of each paradigm level and its function in the algorithm follows.

Table 6. Purposes and Research Methods for Levels of Paradigm

	Level	Purpose	Research Method
I.	Theory	Identification and Articulation of Theoretical Base or Rationale	Analytic-Synthetic Review
II.	Reality	Application of Theory to Real World	Ethnographic Methods
III.	Clarification	Formulation of General Questions Related to How Subjects Think	Non-structured Interview
IV.	Refinement	Development of Specific Hypotheses Related to How Subjects Think	Structured Interview
V.	Intervention	Development of an Instructional Design	Teaching Experiment
VI.	Demonstration	Confirmation or Negation of Statistical Hypotheses Relative to an Instructional Design	Group Experimental Designs

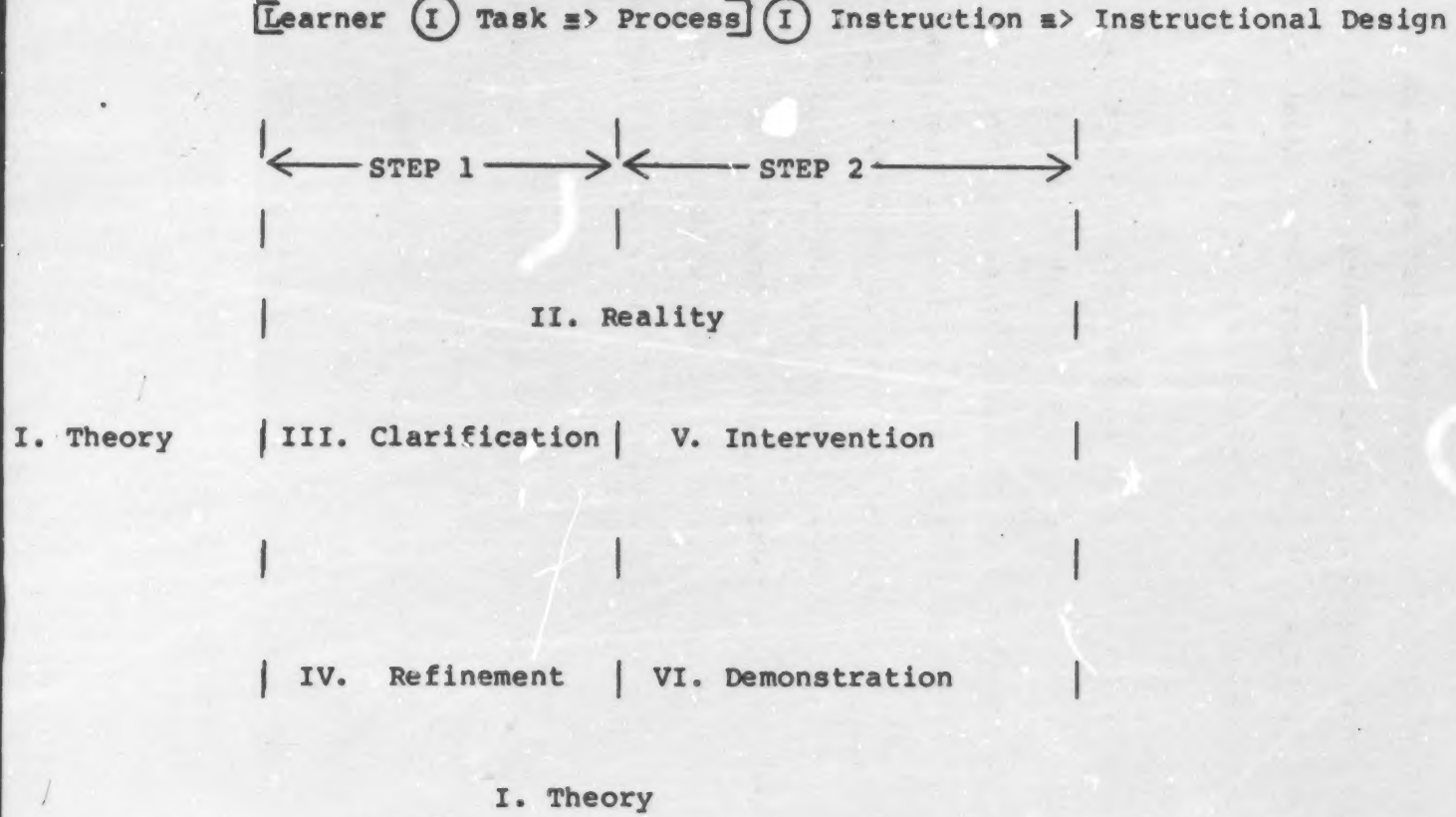

Figure 3. Research Methods in Relation to Algorithmic Steps

I. Theory

Theory stimulates and dominates research from planning through interpretation of results. Theoretical rationales or conceptual frameworks, even loosely formulated ones, serve to establish the context within which judgements are made about what to study and under what conditions. Without theory, research lacks direction (Kaplan, 1964).

The purpose of Level I (Theory) is to identify and articulate the initial theoretical base or rationale underlying the research. Analysis and synthesis of the literature is necessary to achieve this purpose. Specifically, the researcher must analyze relevant theory, its assumptions and constructs and/or propositions, to determine its internal consistency and completeness. Related research studies must also be analyzed. The theory or rationale underpinning research either will evolve from a synthesis of existing theories or points of view, or will be an existing rationale.

In the system being examined, theory would focus on learner, task, process and instructional attributes singly and in combination. Taxonomies for classifying attributes within these four categories would be reviewed and analyzed, and then synthesized if necessary. Research studies examining relationships between and/or among the four components would be analyzed in an attempt to develop some propositions or statements about learning and/or instruction. The review of theories and research

might be limited in scope to a given type of learner or task, or instructional variable.

Beilin's work, Linguistic, Logical, and Cognitive Models for Learning Mathematical Concepts (1975), represents a type of activity intended for Level I. Beilin analyzes the nature of mathematics and the nature of the learner of mathematics within the context of three theoretical models. He develops a rationale to support the notion that "no full comprehension of the interaction between developing cognition and the complex fields of mathematics is likely without a conceptualization of both mathematics as a set of logical, linguistic, and computational theories and the learner as a complex of developing cognitive structures and processes" (Beilin, 1975, p. 32).

II. Reality

The purpose of research efforts at Level II (Reality) is to contrast theory and/or propositions about learning and instruction with reality. The data required to make the contrast are collected using ethnographic methods. Efforts at this level also afford the researcher the opportunity to identify and informally assess how other systems in the environment affect or influence learning and instruction. Other systems might reasonably include components related to local political climate, socioeconomic status, organizational structure of school and/or classroom, classroom climate, teachers, parents and culture.

Ethnographic methodology involves direct observation of phenomena in natural settings. Two assumptions underlie this anthropological method (Wilson, 1977). First, human behavior is influenced by settings; the same behavior or process would yield different findings when observed in a laboratory rather than in a natural setting. Subjects in contrived settings often are suspicious of the researcher's intent and interested in exhibiting expected behavior. Second, to understand meanings of behavior one must attempt to view the context from the subjects' perspectives.

There are two types of direct observation, systematic and unsystematic. Systematic observation involves the use of observation systems derived from theory and controls such factors as observer reliability. In unsystematic observation the observer, sometimes referred to as a participant observer, is guided by theory but does not use an observation system; qualitative judgments about observed behavior and/or events are more likely to be made. Data are recorded in narrative form and categorized at a later time. Unsystematic observation is better for identifying unexpected behavior and/or events, and for understanding the meanings of behavior and/or events. However, the amount of data can become overwhelming.

In planning an ethnographic study, consideration is given to description of setting, entry and role of the observer, data collection, and data analysis (Wilson, 1977). For example, an

observer must monitor his or her entry into a setting and continually assess how he or she is viewed by subjects to help document the occurrence of natural behavior; decide about what or whom to observe, the type of data to collect, and observation schedules (sampling); give thought to how to organize/categorize data best to make comparisons.

At this paradigm level the researcher observes in a natural setting an individual or types of individuals interacting with content. It is suggested that initial observations be general in nature with subsequent observations becoming more focused and content specific. In analyzing results it is important to identify behavior or events supported by theory, in conflict with theory and/or not accounted for by theory. It is also important to identify and informally assess other systems in the environment affecting learning and instruction.

III. Clarification

In Level III (Clarification) the researcher seeks to clarify and explore agreements and/or discrepancies between theory or propositions and reality using non-structured interviews. General questions are formulated about how subjects think. Before conducting the interview, learner and task attributes (input flows) must be carefully specified and measured where possible.

There are at least two types of non-structured interviews, free association and focused (Sjoberg & Nett, 1968). The free-

association interview is more a therapeutic device than a research method. This technique is associated with the work of Sigmund Freud and Carl Rogers. In the focused interview (Merton, Fiske, & Kendall, 1956) the subject has experienced or is experiencing certain conditions that have been analyzed from a theoretical perspective. The interviewer is interested in ascertaining the subject's interpretations, definitions, and/or descriptions of the experience(s) in relation to theory. The focused interview is of most importance for Level III research.

Specifically, the non-structured interview develops around a carefully designed protocol. The questions or tasks in the protocol directly reflect the specific aspects of theory or the propositions being investigated. However, the interviewer is free to deviate from the protocol to probe subject responses in attempting to determine how a subject thinks. Chance, intuition, and imagination play important roles in probing responses; no two interviews proceed in the same way (Romberg & Uprichard, 1977). The nature of the interview requires that the effective interviewer have good interpersonal process skills to facilitate communication. Finally, the non-structured interview is lengthy and generally implemented with a small but carefully selected sample.

The actual interview represents the interaciton of learner and task components (Step 1). To ensure accuracy, interviews should be tape recorded (preferably videotaped) and transcribed.

In analyzing interviews the researcher must attempt to organize responses such that patterns or categories begin to emerge. Categories or patterns help give meaning to responses and establish a context from which general questions related to theory or specific propositions (how subjects think) evolve. Questions should focus on process attributes such as executive control, encoding or recall.

IV. Refinement

General questions related to theory, how subjects think, are refined to specific hypotheses at Level IV (Refinement). Data influencing this researcher task are collected by means of structured interviews.

The structured interview and non-structured interview are similar in that both develop around a carefully designed protocol comprised of questions or tasks which directly reflect theory and the goal of the interviewer is to determine how subjects think. The interviews differ in how they are presented. In the structured interview the protocol is presented in a systematic manner; each subject is asked the same questions or tasks with no format deviations (Romberg & Uprichard, 1977). This type interview can involve one-to-one interaction between interviewer and subject, or the administration of questionnaires or tests. Researchers employing structured interviews are more interested in supporting previously identified patterns of behavior and less concerned with discovering new patterns. Interview responses are

classified in predetermined categories; various statistical techniques can be used to analyze data. The standardization of structured interviews allows for larger sample sizes than possible with non-structured interviews.

Some guidelines proposed by Sjoberg and Nett (1968) for developing structured interview questions and tasks include:

1. Don't assume too much knowledge on part of subject.
2. Employ simple, straightforward vocabulary and style.
3. Use questions/tasks from other studies to establish baseline data.
4. Cross-check questions/tasks--measure same concept more than once.

Analysis of structured interview data has been criticized because pre-determined categories are imposed on subject responses. It is thought that this practice makes it difficult to comprehend the complexities of human interaction and oversimplifies reality (Sjoberg & Nett, 1968). However, this criticism is less valid when non-structured interviews (Level III - Clarification) precede structured interviews.

The use of structured interviews in step 1 of the system's algorithm is restricted to individual interviews. The interviewer must observe the interaction between learner and tasks. Again, the goal of Level IV is to refine general questions about theory (in this case, how subjects think) to hypotheses. It is best if individual interviews are recorded electronically.

V. Intervention

In Level V (Intervention) specific hypotheses about how subjects think are used in designing instruction and the instruction evaluated by means of a teaching experiment (Krutetskii, 1976; Vygotsky, 1962). Research at this level focuses on the interaction of process and instructional attributes (Step 2).

The teaching experiment is characterized by its specificity. With this method the investigator or teacher interacts with a small but select group of subjects around a carefully designed instructional sequence or protocol. Progress through the sequence or protocol is systematically monitored by means of observations (videotapes), interviews and analysis of written work. Instruction is altered on the basis of teacher-student interactions. Of primary interest are the instructional conditions under which subjects acquire new knowledge rather than what they know or how they think (Romberg & Uprichard, 1977). Constructing models about how subjects think relative to specific content is addressed at preceding paradigm levels. The teaching experiment as used in this level of the paradigm is considered somewhat specialized. However, others have interpreted the teaching experiment in a similar manner. For example, Kantowski (1979) states, "The objective of the teaching experiment is to construct methods of instruction in accordance with a hypothesis outlined on the basis of observations and preliminary experi-

ments, and to study the effects of those methods of instruction" (p. 136).

Decisions about what types of data to collect in a teaching experiment are influenced by theory. Analysis of data is used to refine the instructional design and to determine its overall effectiveness. The most important outcome of the teaching experiment is the "finished" design or protocol. Also, a second opportunity to informally assess other systems in the environment affecting learning and instruction is provided at this paradigm level.

VI. Demonstration

Research efforts in Level VI (Demonstration) are guided by a logic of proof; specific statistical hypotheses are confirmed or denied using group experimental designs. Hypotheses relate directly to the instructional design derived from the teaching experiment; the efficiency of the instructional design is demonstrated via comparison.

Group experimental designs and their accompanying statistical analyses are well documented in the literature (Hays, 1973; Winer, 1971). Typical characteristics of these designs are random or stratified samples, random assignment of subjects, pre-set treatments and control groups. Data are collected in a systematic manner and analyzed using inferential statistics; null hypotheses are supported or rejected at given probability levels. Results are generalizable to the population sampled. Deviations

from design and/or statistical assumptions can threaten internal and/or external validity.

Implementation of group design research can create logistical and expense problems. Both problems are worth solving if research findings have the potential to impact educational practice. Degree of impact is dependent upon the theoretical underpinnings of research hypotheses. Research associated with supporting paradigm levels (I-V) better assures the legitimacy of statistical hypotheses tested in group experimental designs. Still, many questions are being raised about the necessity for this type research in education.

Comments

In planning research on learning and instruction consideration must be given to describing variables and variable interactions, and methods to best study them. The proposed system and related research paradigm provides a possible skeletal structure for organizing this work. In addition, its use stimulates the planning of successive studies.

The lack of specificity in describing variables and procedures (Baker, 1969) and the absence of continuity of studies (Goodlad, 1969) has impeded progress in educational research. Using the system and paradigm in conjunction with educational technology can reverse this trend. Specifically, research can be improved by:

1. developing and using common taxonomies to code input flows, interactions, and related outputs or products.

2. standardizing the measurement of variable attributes, as well as observation procedures, interview techniques and protocals. Indeed, some tasks can be programmed for the microcomputer.
3. storing data collected by individual investigators on computer discs and then transfering the data to a mainframe computer at a predetermined site. Once centralized, the data can be used to make type and cross level generalizations or predictions about learning and instruction.

Is this a dream? Maybe. But the pursuit of new knowledge requires dreamers.

Discovering how a person thinks or learns, and using that knowledge to design instruction is at the crux of the educational enterprise. The proposed system and related research paradigm represents an attempt to bring a diagnostic/prescriptive focus to research on learning and instruction in order to better achieve this purpose. They are presented at the risk of oversimplifying complex and dynamic phenomena or events.

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